A more complex example on Predicate Logic inference for the class.

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1 The setting

We will give a more complex example on propositional logic inference for the class to study. It's a modified version of the example we did in class. Table 1 details all propositional logic rules that we can use on ground predicates (or ground statements corresponding to combinations of those ground predicates) and table 2 outlines the "lifted" versions of modus ponens and modus tollens that predicate logic uses.

Modus Ponens	Modus Tollens	Disjunctive addition	Conjunctive addition	Conjunctive Simplification
p $p \Rightarrow q$ $\therefore q$	$\sim q$ $p \Rightarrow q$ $\therefore \sim p$	p $\therefore p \vee q$	p, q $\therefore p \wedge q$	$p \wedge q$ $\therefore p, q$
Disjunctive syllogism	Hypothetical syllogism	Unit Resolution	Resolution	
$p \vee q$ $\sim p$ $\therefore q$	$p \Rightarrow q$ $q \Rightarrow r$ $\therefore p \Rightarrow r$	$p \vee q$ $\sim q$ $\therefore p$	$p \vee q$ $\sim q \vee z$ $p \vee z$	

Table 1: Propositional Logic inference rules we can use.

Universal Modus Ponens	**Universal Modus Tollens**
$(\forall x \in D)P(x) \Rightarrow Q(x)$	$(\forall x \in D)P(x) \Rightarrow Q(x)$
$P(A)$ for some A $\in D$	$\sim Q(A)$ for some A $\in D$
$Q(A)$	$\sim P(A)$

Table 2: Predicate Logic inference rules we can use.

2 Predicates

Table 3 describes the predicates that we will use.

Predicate	**Meaning**
$Midterm(n, s, g)$	The grade of student s in midterm number n (1 or 2) was g (A, B or C).
$Final(s, g)$	The grade of student s in midterm in the final was g (A, B, or C).
$Present(s)$	Student s was consistently present in lecture.
$Studies(s, l)$	Student s studies in mode l ($Lazily$, $Well$ or $Hard$)
$Passes(s, g)$	Student s passed the course with grade g.
$Fails(s)$	Student s failed the course.

Table 3: The predicates we will use when translating English language statements to Predicate Logic statements.

3 Knowledge base

3.1 In English

(i) **Every** student who studies hard will get As in both midterms and **at least** a B in the final.

(ii) **Any** student who is consistently present in lecture will score **at least** a B in **both** midterms and **at least** a C in the final.

(iii) **One** will pass the course with a B **if, and only if**, one scores **at least** a C in the final, and a B **or better** on **either** midterm.

(iv) **Any** student who studies well or hard will score **at least** a B in **both** midterms **and** the final.

(v) **One cannot** pass and fail the course at the same time.

3.2 In Predicate Logic

(i) $(\forall s) Studies(p, Hard) \Rightarrow Midterm(1, s, A) \wedge Midterm(2, s, A) \wedge (Final(s, B) \vee Final(s, A))$

(ii) $(\forall s) Present(s) \Rightarrow (Midterm(1, s, B) \vee Midterm(1, s, A)) \wedge (Midterm(2, s, B) \vee Midterm(2, s, A)) \wedge (Final(s, C) \vee Final(s, B) \vee Final(s, A))$

(iii) $(\forall s)(Final(s, C) \vee Final(s, B) \vee Final(s, A)) \wedge (Midterm(1, s, B) \vee Midterm(1, s, A) \vee Midterm(2, c, B) \vee Midterm(2, c, A)) \Leftrightarrow Passes(s, B)$

(iv) $(\forall s) Studies(s, Well) \vee Studies(s, Hard) \Rightarrow (Midterm(1, s, B) \vee Midterm(1, s, A)) \wedge (Midterm(2, s, B) \vee Midterm(2, s, A)) \wedge (Final(s, B) \vee Final(s, A))$

(v) $(\forall s, g) \sim (Passes(s, g) \wedge Fails(s))$

4 The input and the derivation goal

Given the statement "Trisha is a student consistently present in lecture who studies well", we want to derive that she is will pass the course (with some grade).

5 The proof

5.1 Formulation of input and derivation goal

We translate the input into predicate logic to get:

$$Studies(Trisha, Well) \wedge Present(Trisha) \tag{1}$$

Our goal is to derive:

$$(\exists g) Passes(Trisha, g). \tag{2}$$

5.2 Derivation steps

We will use the information that Trisha studies well in order to "drive" our inference through the parts of our knowledge base that utilize this information. First of all, we denote that because of the definition of the bi-conditional, the rule (iii) can be written as:

$$\begin{aligned} &\Big((\forall s)\Big(Final(s,C) \lor Final(s,B) \lor Final(s,A)\Big) \land \Big(Midterm(1,s,B) \lor Midterm(1,s,A) \\ &\lor Midterm(2,c,B) \lor Midterm(2,c,A)\Big) \Rightarrow Passes(s,B)\Big) \bigwedge \Big((\forall s,B) Passes(s,B) \\ &\Rightarrow \Big(Final(s,C) \lor Final(s,B) \lor Final(s,A)\Big) \land \Big(Midterm(1,s,B) \\ &\lor Midterm(1,s,A) \lor Midterm(2,c,B) \lor Midterm(2,c,A)\Big)\Big) \end{aligned} \tag{3}$$

and then we can use the rule of conjunctive simplification to break (3) into its constituent parts:

$$\begin{aligned} &(\forall s)\Big(Final(s,C) \lor Final(s,B) \lor Final(s,A)\Big) \land \Big(Midterm(1,s,B) \\ &\lor Midterm(1,s,A) \lor Midterm(2,c,B) \lor Midterm(2,c,A)\Big) \Rightarrow Passes(s,B) \end{aligned} \tag{4}$$

and

$$\begin{aligned} (\forall s,B) Passes(s,B) \Rightarrow &\Big(Final(s,C) \lor Final(s,B) \lor Final(s,A)\Big) \land \Big(Midterm(1,s,B) \\ &\lor Midterm(1,s,A) \lor Midterm(2,c,B) \lor Midterm(2,c,A)\Big) \end{aligned} \tag{5}$$

Our input ((1)) can also be analyzed into its constituent parts by the law of conjunctive simplification:

$$Studies(Trisha, Well) \tag{6}$$

and

$$Present(Trisha) \tag{7}$$

The law of disjunctive addition can be used to expand (6) into:

$$Studies(Trisha, Well) \vee Studies(Trisha, Hard) \tag{8}$$

and then we can use Universal Modus Ponens on (8) and (iv) to derive:

$$\begin{aligned} &Midterm(1, Trisha, B) \vee Midterm(1, Trisha, A)) \\ &\wedge (Midterm(2, Trisha, B) \vee Midterm(2, Trisha, A)) \wedge (Final(Trisha, B) \vee Final(Trisha, A)) \end{aligned} \tag{9}$$

We can subsequently use conjunctive simplification on (9) to break into its three constituent parts:

$$Midterm(1, Trisha, B) \vee Midterm(1, Trisha, A)) \tag{10}$$

$$Midterm(2, Trisha, B) \vee Midterm(2, Trisha, A)) \tag{11}$$

$$Final(Trisha, B) \vee Final(Trisha, A) \tag{12}$$

(12) can be expanded using the law of disjunctive addition to derive:

$$Final(Trisha, C) \vee Final(Trisha, B) \vee Final(Trisha, A) \tag{13}$$

(10) and (11) can be combined via the law of disjunctive addition to derive:

$$\Big(Midterm(1, Trisha, B) \vee Midterm(1, Trisha, A))\Big) \bigvee \Big(Midterm(2, Trisha, B) \vee Midterm(2, Trisha, A))\Big) \quad (14)$$

which, of course, because of the associativity of disjunction, can be simplified as:

$$Midterm(1, Trisha, B) \vee Midterm(1, Trisha, A) \vee Midterm(2, Trisha, B) \vee Midterm(2, Trisha, A) \quad (15)$$

We're almost there. (13) and (15) can be combined via the law of conjunctive addition to yield:

$$\Big(Final(Trisha, C) \vee Final(Trisha, B) \vee Final(Trisha, A)\Big) \bigwedge \Big(Midterm(1, Trisha, B) \\ \vee Midterm(1, Trisha, A) \vee Midterm(2, Trisha, B) \vee Midterm(2, Trisha, A)\Big) \quad (16)$$

And we are now ready to combine (16) with (4) to yield:

$$Passes(Trisha, B) \quad (17)$$

Are we done? **Not quite!** Our desired conclusion can be reached by applying the law of existential generalization on (17):

$$(\exists g) Passes(Trisha, g) \quad (18)$$

Since (18) matches the stated goal at (2), we have concluded our proof. As a piece of homework, you can try to see if using the other piece of information in your input, the fact that Trisha is consistently present in lecture, can help you with respect to deriving our desired conclusion.