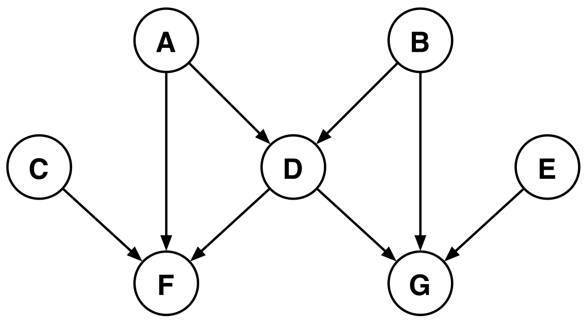
CSE 150. Assignment 3 *Summer 2016*

Out: *Tue Jul 05*
Due: *Fri Jul 08 (outside CSE 3214, by 10 am)*
Supplementary reading: KN, Ch 2.4-2.5.

3.1 Conditional independence

For the belief network shown below, indicate whether the following statements of marginal or conditional independence are **true (T)** or **false (F)**.



________ $P(A, B|D) = P(A|D)\,P(B|D)$

________ $P(G|D) = P(G|F, D)$

________ $P(G|B, D) = P(G|A, B, D)$

________ $P(D|A, B, F, G) = P(D|A, B, C, E, F, G)$

________ $P(C, E|A, D, F, G) = P(C|A, D, F)\,P(E|A, D, G)$

________ $P(A, B|F, G) = P(A|F, G)\,P(B|F, G)$

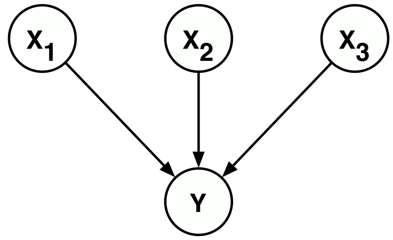
________ $P(A, B|D, F, G) = P(A|D, F)\,P(B|D, G)$

________ $P(B|D, F, G) = P(B|C, D, F, G)$

________ $P(A, G|D) = P(A|D)\,P(G|D)$

________ $P(C, D, E) = P(C)\,P(D)\,P(E)$

3.2 Noisy-OR



(a) **Conditional probability table**

Complete the noisy-OR conditional probability table for the belief network shown above. (The missing values in the table are determined by the ones shown.)

X_1	X_2	X_3	$P(Y=1\|X_1, X_2, X_3)$
0	0	0	0
1	0	0	$\frac{1}{5}$
0	1	0	
0	0	1	
1	1	0	$\frac{2}{5}$
1	0	1	
0	1	1	
1	1	1	$\frac{1}{2}$

(b) **Qualitative reasoning**

Suppose that each node X_i in this model has some finite, non-zero prior probability to be either zero or one; namely $0 < P(X_i\!=\!1) < 1$. Sort the following probabilities from smallest to largest:

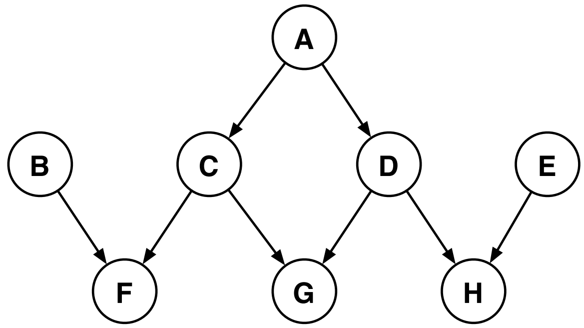
$$P(X_1\!=\!1)$$
$$P(X_1\!=\!1|Y\!=\!0)$$
$$P(X_1\!=\!1|Y\!=\!1)$$
$$P(X_1\!=\!1|Y\!=\!1, X_2\!=\!0, X_3\!=\!0)$$
$$P(X_1\!=\!1|Y\!=\!1, X_2\!=\!1, X_3\!=\!1)$$

For this part, you are **not** required to compute numerical values for these probabilities, only to sort them in ascending order. In fact, this part can be completed independently of the values in part (a).

(c) **Quantitative reasoning**

Suppose that $P(X_i = 1) = \frac{1}{2}$ for $i \in \{1, 2, 3\}$. Compute numerical values for the probabilities in part (b) from these priors and your answers in part (a). Do these values match your previous ordering?

3.3 Subsets

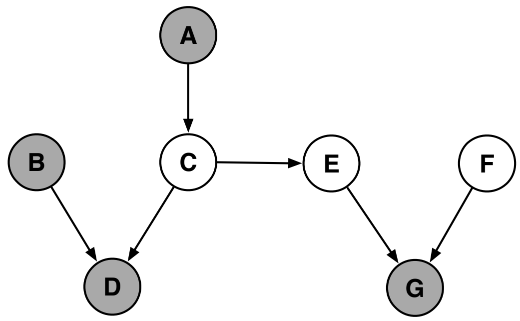


Consider the following statements of marginal or conditional independence for the belief network shown above. Indicate the largest subset of nodes $\mathcal{S} \subset \{A, B, C, D, E, F, G, H\}$ for which each statement is true. Note that one possible answer is the empty set $\mathcal{S} = \emptyset$ or $\mathcal{S} = \{\}$ (whichever notation you prefer). The first has been done as an example.

$P(A\|C)$	=	$P(A\|\mathcal{S})$	$\mathcal{S} = \{B, C, E, F\}$
$P(A)$	=	$P(A\|\mathcal{S})$	________________
$P(D)$	=	$P(D\|\mathcal{S})$	________________
$P(D\|A)$	=	$P(D\|\mathcal{S})$	________________
$P(D\|A, G, H)$	=	$P(D\|\mathcal{S})$	________________
$P(D\|A, C, E, G, H)$	=	$P(D\|\mathcal{S})$	________________
$P(F)$	=	$P(F\|\mathcal{S})$	________________
$P(G\|F)$	=	$P(G\|\mathcal{S})$	________________
$P(F, H)$	=	$P(F, H\|\mathcal{S})$	________________
$P(B, E)$	=	$P(B, E\|\mathcal{S})$	________________
$P(H\|B)$	=	$P(H\|\mathcal{S})$	________________

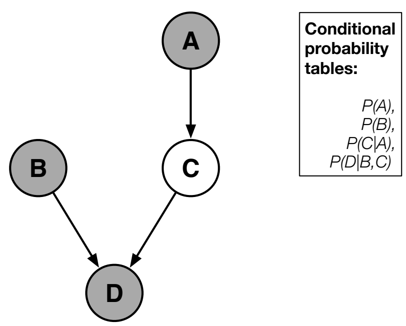
3.4 Inference in a polytree

Consider the belief network shown below. In this problem you will be guided through an efficient computation of the posterior probability $P(F|A, B, D, G)$. You are expected to perform these computations *efficiently*—that is, by exploiting the structure of the DAG and not marginalizing over more variables than necessary. *Justify your steps briefly for full credit.*



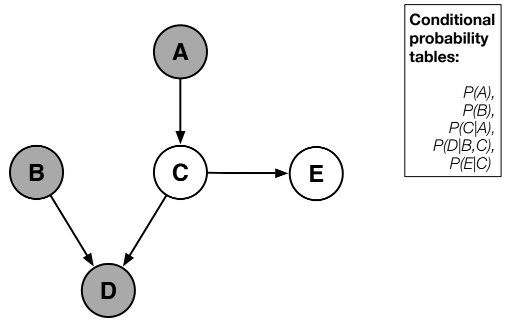
(a) **Bayes rule**

Consider just the part of the belief network shown below. Show how to compute the posterior probability $P(C|A, B, D)$ in terms of the conditional probability tables (CPTs) for these nodes—i.e., in terms of $P(A)$, $P(B)$, $P(C|A)$, and $P(D|B, C)$.



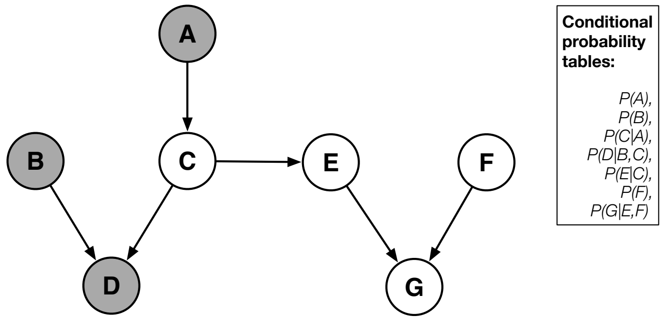
(b) **Marginalization**

Consider just the part of the belief network shown below. Show how to compute the posterior probability $P(E|A, B, D)$ *in terms of your answer from part (a)* and the CPTs of the belief network.



(c) **Marginalization**

Consider the belief network shown below. Show how to compute the posterior probability $P(G|A, B, D)$ *in terms of your answer from part (b)* and the CPTs of the belief network.



(d) **Explaining away**

Finally, show how to compute the posterior probability $P(F|A, B, D, G)$ *in terms of your answer from parts (b,c)* and the CPTs of the belief network.