PHY 354S: PROBLEM SET #1 **Due: Friday, January 27, 2017**

Do all problems. The starred question will be marked in detail. The other problems will be graded out of 2 to give you an incentive to do them: partial solutions will receive 1 mark; correct or almost correct solutions will receive full marks.

1. A particle is subject to a potential $V(x) = -kx$, where k is a constant. The mass travels from $x = 0$ to $x = a$ in a time interval t_0. Assume the motion of the particle can be expressed in the form $x(t) = A + Bt + Ct^2$. Find the values of A, B and C such that the action is a minimum.

2. A bead of mass m is constrained to move along the smooth conical spiral shown in the figure. At any height z, the coordinates on the spiral are given by $\rho = az$ and $\phi = -bz$ for constants a and b. Find the Lagrangian and equations of motion of the bead in terms of the height z of the bead.

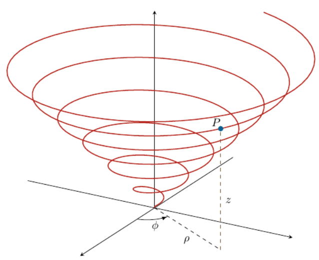

Figure 1: Problem 2

3. (Landau & Lifshitz, Problem 3 (a))

   Consider a simple pendulum of mass m and length L. Write down the Lagrangian and derive the equations of motion for the angle θ between the pendulum and the vertical axis if the pivot moves uniformly on a vertical circle of radius R rotating with constant frequency ω (as shown in the figure). (HINT: for this type of question, often the most straightforward way to find the velocity of the mass is to write down an expression for its Cartesian coordinates at time t in terms of the generalized coordinates, and then differentiate.)

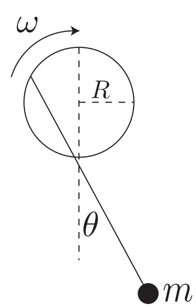

Figure 2: Problem 3.

*4. (a) A circular wire hoop rotates in the horizontal plane at constant angular velocity ω about a vertical axis through the point A in the Figure (the figure is shown viewed from above). A bead of mass m is threaded on the hoop and free to move around it, with its position specified by the angle ϕ shown in the Figure. Find the Lagrangian for this system using ϕ as your generalized coordinate. Show that the bead oscillates about the point B exactly like a simple pendulum. What is the frequency of these oscillations if their amplitude is small?

(b) Consider a bead of mass m sliding on a wire that is bent in the shape of a parabola and is being spun with constant angular velocity ω about its vertical axis, as shown in the Figure. Take the equation of the parabola to be $z = k\rho^2$ for some constant k. Find the Lagrangian in terms of the generalized coordinate ρ, and find the equation of motion of the bead. Are there any points of equilibrium (values of ρ at which the bead can remain fixed, without sliding up or down the spinning wire)? Discuss the stability of any equilibrium positions you find, as a function of the frequency ω. (Recall: an equilibrium position is stable if there is a restoring force for small perturbations about the equilibrium point, otherwise it is unstable).

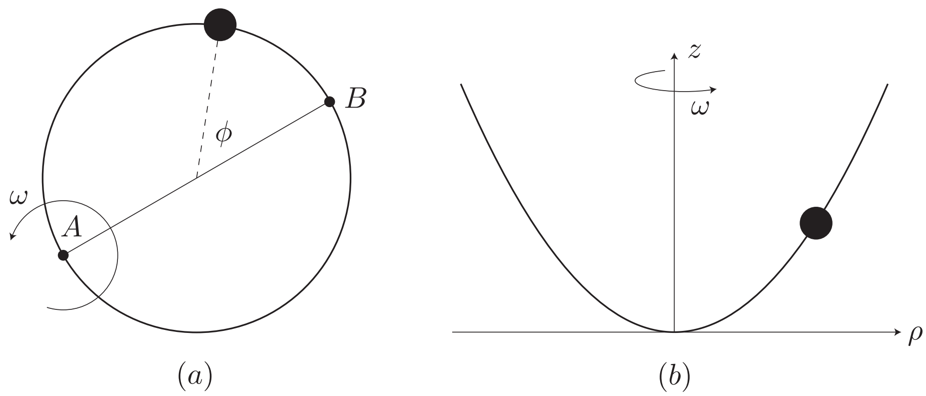

Figure 3: Problem 4