Sets II

Last time we discussed the Axioms of Extension, Specification, Unordered Pairs, and Unions.

Some more aioms of set theory

Powers For each set there exists a collection of sets that contains among its elements all the subsets of the given set. That is, if A is a set, then $\mathcal{P}(A) = \{B : B \subset A\}$ is a set.

Infinity There exists a set containing 0 and containing the successor of each of its elements.

Regularity Every non-empty set contains an element that is disjoint from itself.

Notes:

- We define 0 to be the empty set $\varnothing$. (Everything in our world is a set now.)
- The *successor* of a set x is the set $x^+ = x \cup \{x\}$.
- The Axiom of Infinity replaces the assumption that "a set exists" from the previous worksheet.
- The Axiom of Regularity is not in Halmos' book, and much of set theory can be developed without it, but I am including it here because many of you want to discuss the issue of a set being an element of itself.
- We are omitting the Axiom of Replacement, which says that the image of a set under a function is a set. We will discuss functions in much more details later.

1. Use the Axioms of Regularity (and Unordered Pairs and Specification) to prove that a set cannot be an element of itself.
2. Prove that "There is no Universe" with or without using the Axiom of Regularity.
3. Explain why the intersection of an empty collection is undefined. But if we restrict our attention to a collection of subsets of a fixed set E, then the definition of intersection can be modified to make sense of the empty-intersection. How?
4. Write down the successor of 0, and its successor, and its successor, etc. five times. You can give them "natural" names.