Bone: *fracture mechanics*

Sources of information for fracture mechanics stuff:
Ch5 Martin & Burr fracture mechanics
Callister supplement on fracture
(maybe good for those who are unfamiliar or rusty on fracture)

Some relevant and useful papers (these and others by RO Ritchie)

A fracture mechanics and mechanistic approach to the failure of cortical bone

R. O. RITCHIE[1], J. H. KINNEY[2], J. J. KRUZIC[1,3] and R. K. NALLA[1]

© 2005 Blackwell Publishing, Ltd. *Fatigue Fract Engng Mater Struct* **28**, 345–371

Mechanistic aspects of fracture and R-curve behavior in human cortical bone ☆

R.K. Nalla[a], J.J. Kruzic[a], J.H. Kinney[b], R.O. Ritchie[a,*]

Biomaterials 26 (2005) 217–231

On the origin of the toughness of mineralized tissue: microcracking or crack bridging?

R.K. Nalla, J.J. Kruzic, and R.O. Ritchie*

Bone 34 (2004) 790–798

Famous Failures – Part 1

The molasses flood of Boston, January 15, 1919

- Molasses was main sweetener, used to make alcoholic beverages and used for munitions manufacture
- Warm day in Boston, pressure buildup
- 27 m dia x 15 m tank fractured,
- >2 mill gal of molasses
- Blocks were flooded 2-3 ft deep
- Buildings and trains swept of tracks or foundations
- Fatigue crack suspected

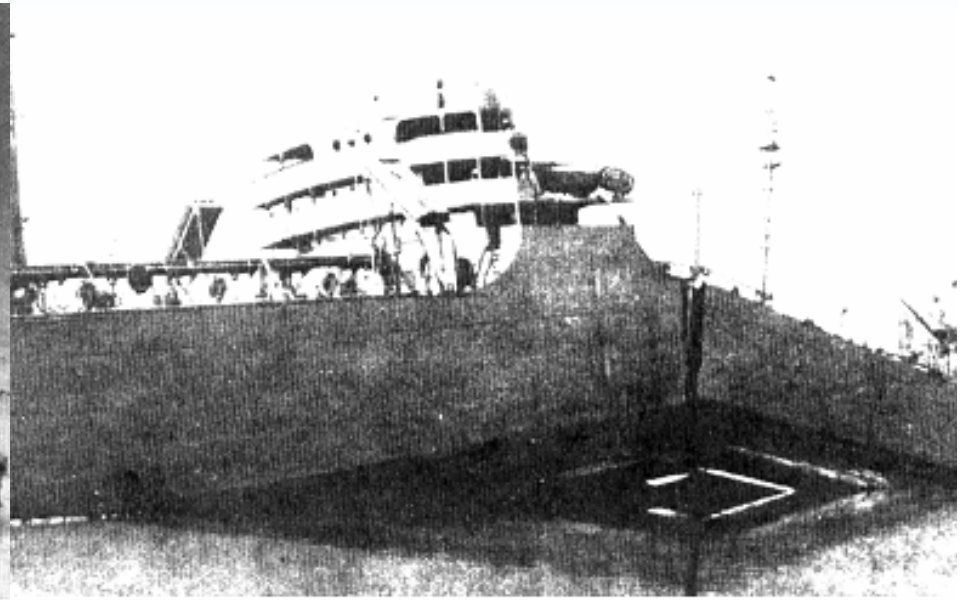
Famous Failures – Part 2
cost human lives, huge economic costs as well

- **1912 The Titanic**

See http://www.tms.org/pubs/journals/jom/9801/felkins-9801.html for discussion of microstructure and properties of steel used in titanic

Reprinted w/ permission from R.W. Hertzberg, "Deformation and Fracture Mechanics of Engineering Materials", (4th ed.) Fig. 7.1(a), p. 262, John Wiley and Sons, Inc., 1996. (Orig. source: Dr. Robert D. Ballard, *The Discovery of the Titanic*.)

- **WWII: Liberty ships**



Reprinted w/ permission from R.W. Hertzberg, "Deformation and Fracture Mechanics of Engineering Materials", (4th ed.) Fig. 7.1(b), p. 262, John Wiley and Sons, Inc., 1996. (Orig. source: Earl R. Parker, "Behavior of Engineering Structures", Nat. Acad. Sci., Nat. Res. Council, John Wiley and Sons, Inc., NY, 1957.)

Origin of fracture mechanics (historical note)

- In 1920's A.A. Griffith noted that fracture strength of materials is ~10-1000x less than the theoretical value.
- Theoretical value estimated to be E/10, based on atomic bonding energies (i.e. cohesive forces between atoms in a material).
- Understanding the mechanisms behind this discrepancy is critical to understanding how to predict and prevent failure of engineering materials.

Origin of fracture mechanics

- In 1920's A.A. Griffith noted that fracture strength of materials is ~10-1000x less than the theoretical value.
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- Understanding the mechanisms behind this discrepancy is critical to understanding how to predict and prevent failure of engineering materials.
- Griffith concluded that the discrepancy may be due to small flaws in the material that exist under normal circumstances.
- **Fracture mechanics is used to understand how the flaws influence failure.**

Why is there interest in fracture toughness in bone?

- *Standard clinical measure of osteoporotic risk of fracture is bone mineral density (BMD).*
- *Evidence that bone mineral density (BMD) cannot be the sole factor for increased fracture risk:*
  - 10-fold increase risk of fracture with age – *independent of BMD (Hui et al).*
  - collagen denaturation can affect fracture toughness of bone (Agrawal et al)
- To understand how different factors affect fracture of bone one must have an understanding of the structure (and microstructure) – property relationships.
- The complicated hierarchical structure of bone makes it difficult to separate what role these structures have in crack initiation, growth, and unstable propagation to failure.
- **RO Ritchie examines cortical bone failure mechanisms and relates the mechanisms to bone structure in his work on fracture of cortical bone.**

Take away these main points on bone fracture mechanics

(but be sure to understand and to study details for exam)

- Bone is tougher in some directions compared to others (anisotropic).
  - What orientations are toughest? Least tough?
  - Why are some orientations tougher?
  - What mechanisms make one direction more tough than another?
- Fracture toughness of bone may not be single valued and may need to be described using R-curve approach.
  - What does an R-curve measure?
  - How is the measure of toughness with an R-curve different from the material property of fracture toughness, K_{IC}?
  - What specific toughening mechanisms cause toughening to increase with increasing crack growth? And *how* do they increase the toughness?
  - How are these types of mechanisms classified?
  - Which ones contribute most to each orientation?

What are the loading conditions under which bone fractures? Fracture can occur

- Monotonically or suddenly (we did this for strength)
  - (i.e. reported strength from previous lectures)

- By creep (we won't do this for bone)
  - constant load over time

- By fatigue (we will do this time permitting)
  - cyclic loading over time

Usually have microcracks that are repaired during remodeling.

- If microcrack accumulation rate > remodeling rate then fatigue failure or stress fracture can occur
- Fracture also depends on orientation of crack relative to loading – and therefore the toughening mechanisms (we will discuss these in more detail)

What are some of the ways to describe toughness/fracture toughness?

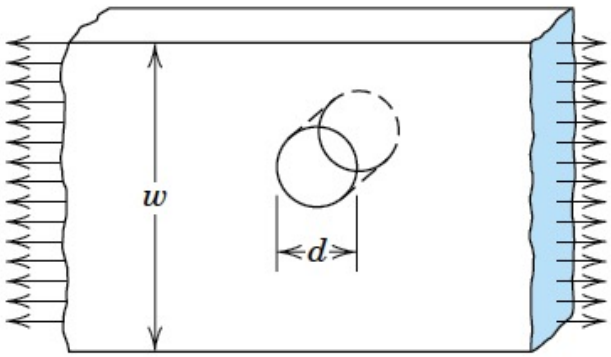
- Work to fracture
  - Work necessary to facture solid and form two surfaces (work/2 x area)
  - Un-notched specimen
  - Toughness in this case is the energy equivalent of fracture strength
  - Depends on pre-existing cracks (including those made during specimen preparation)
  - Depends on specimen size and geometry - Cannot cross-compare data from different studies very well
- Linear elastic fracture mechanics
  - ME 330 approach
  - K_C: critical stress intensity factor (not a material property)
  - K_{IC}: mode I fracture toughness (a material property)
  - Toughness to initiate cracking

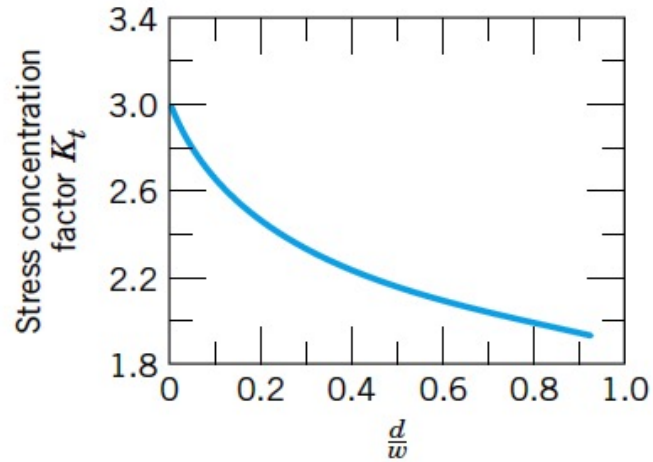
What are some of the ways to describe toughness/fracture toughness? (CONTINUED)

- Crack-resistance curves (R-curves)
  - Toughness of a growing crack
    - i.e. a growing crack stable and has subcritical crack growth
  - Occurs in materials that
    - Deform plastically (metals)
    - Brittle materials that toughen extrinsically (bone, some ceramics)
      - Extrinsic mechanisms shield the crack
      - They have no influence on crack initiation
  - More on R-curves later…
- Nonlinear fracture mechanics and fracture toughness
  - J-integral approach accounts for plasticity
  - We won't do this at all

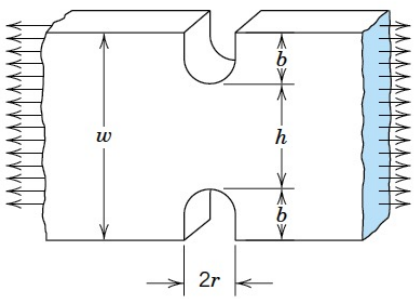
Stress concentration factor, K_t

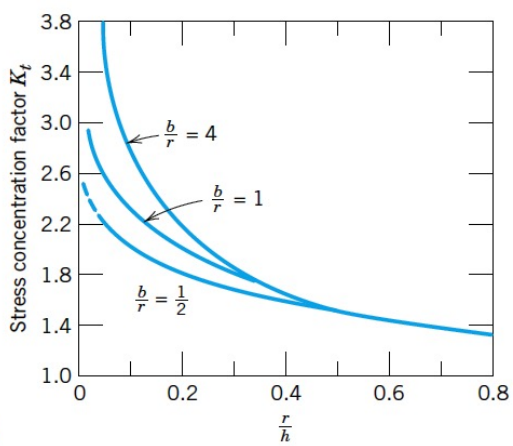
FIGURE 8.2W Theoretical stress concentration factor curves for three simple geometrical shapes. (From G. H. Neugebauer, *Prod. Eng.* NY), Vol. 14, pp. 82–87, 1943.)



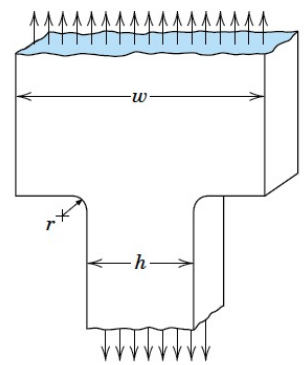


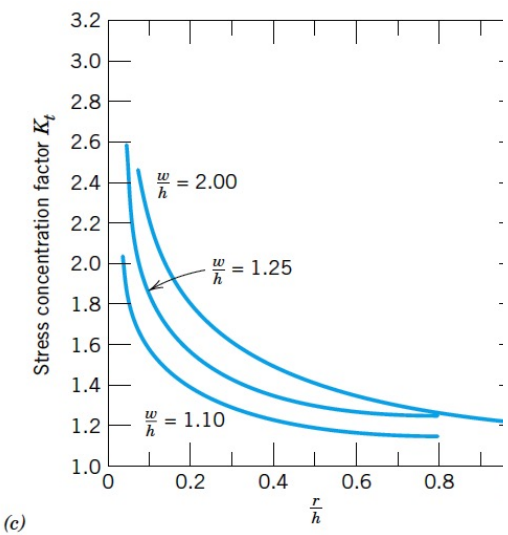
(a)





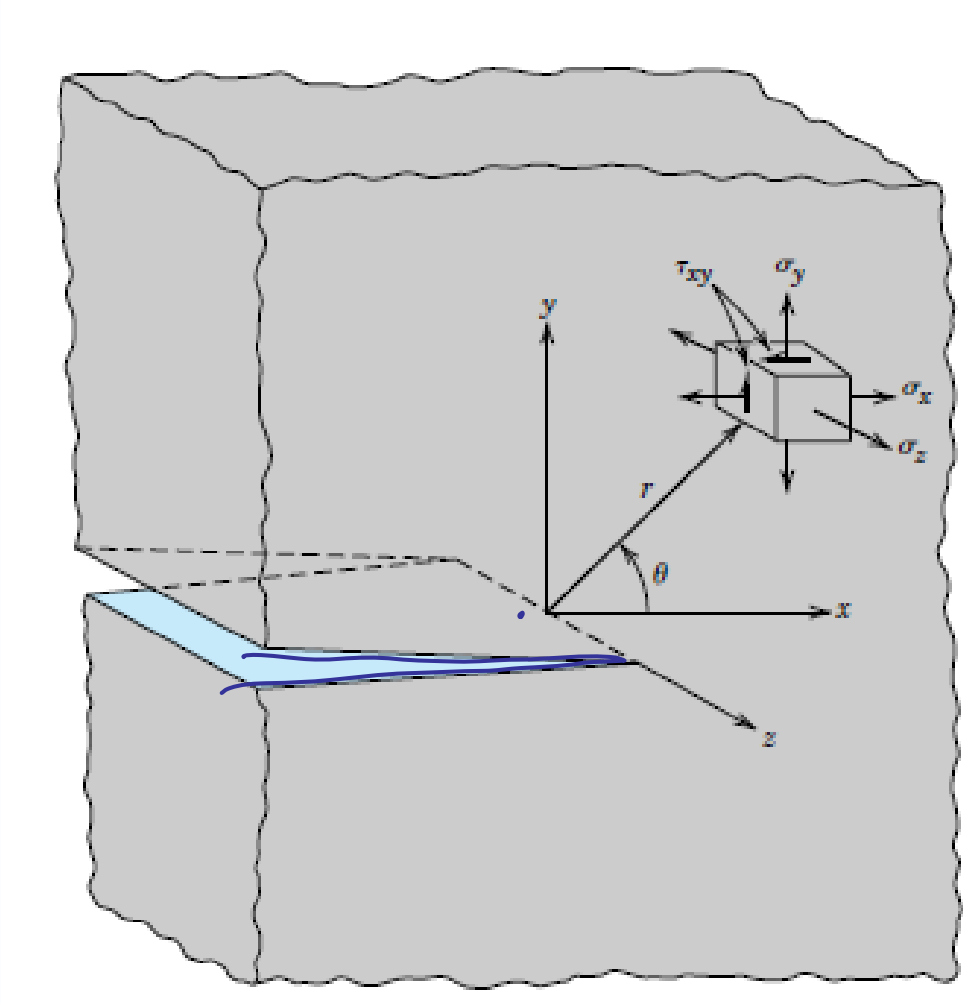
(b)





(c)

The stress field in front of a crack uses K



$$\sigma_x = \frac{K}{\sqrt{2\pi r}} f_x(\theta)$$

$$\sigma_y = \frac{K}{\sqrt{2\pi r}} f_y(\theta)$$

$$\tau_{xy} = \frac{K}{\sqrt{2\pi r}} f_{xy}(\theta)$$

$$f_x(\theta) = \cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$

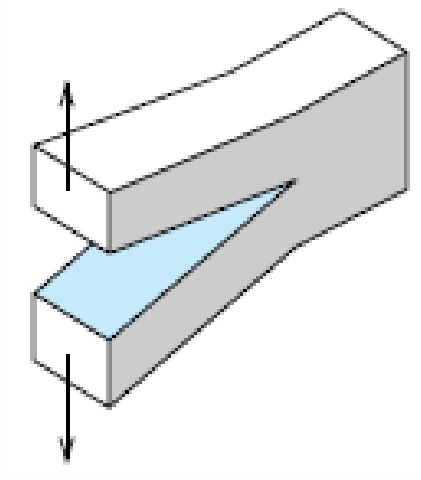
$$f_y(\theta) = \cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$

$$f_{xy}(\theta) = \sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}$$

Review of basic fracture mechanics

Stress intensity factor, K

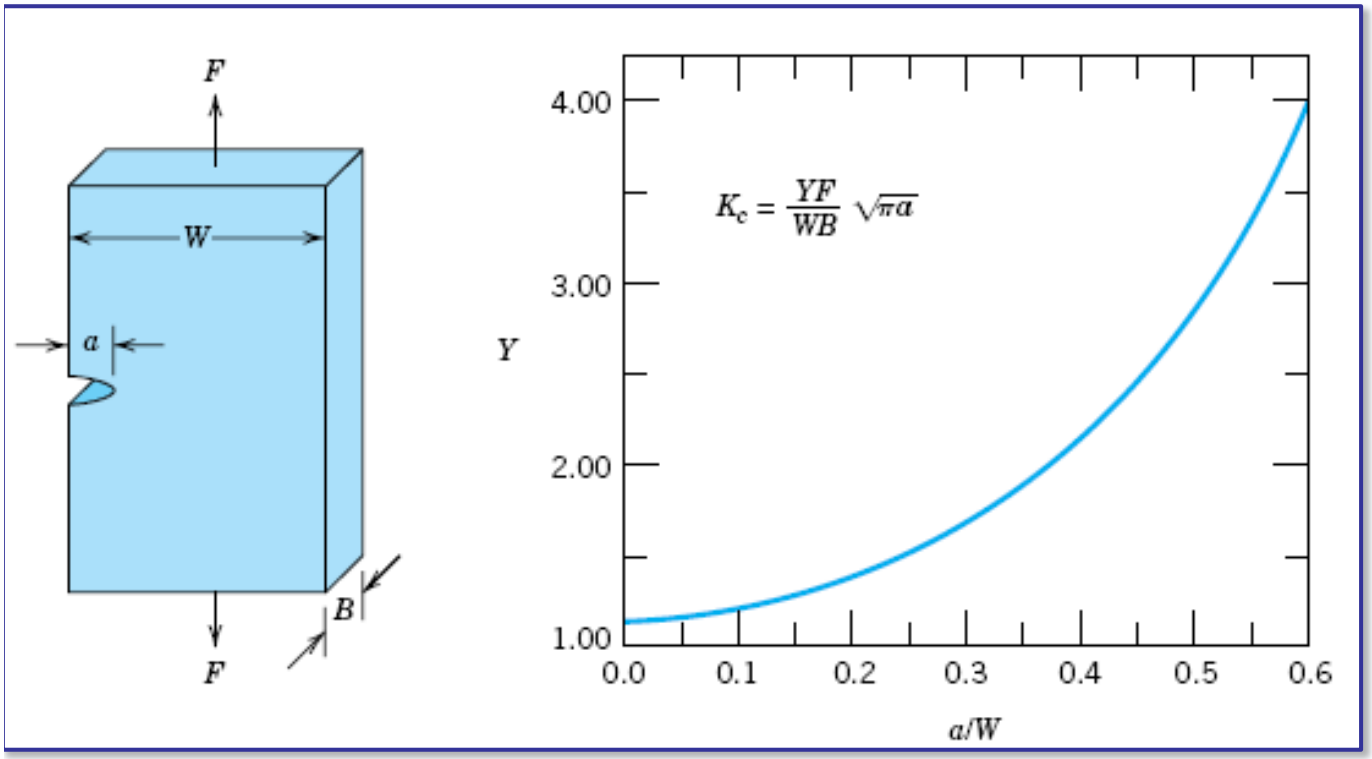
$$K = Y\sigma\sqrt{\pi a}$$

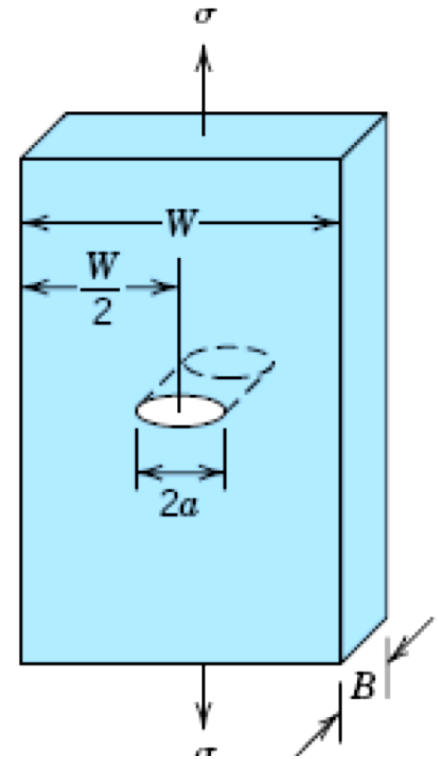


- Kind of a measure of the stress amplification at a crack (stress concentration)
- Used in linear elastic fracture mechanics
  - (inelastic behavior limited to right near crack tip)
- Focus on mode I (shown)
- Stress intensity factor is used to describe the stress and displacement fields in the vicinity of the crack.
  - exact depends on crack shape and deformation mode through Y

What is Y?

$$K = Y\sigma\sqrt{\pi a}$$

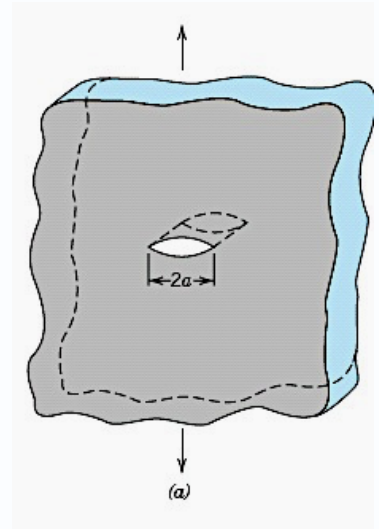




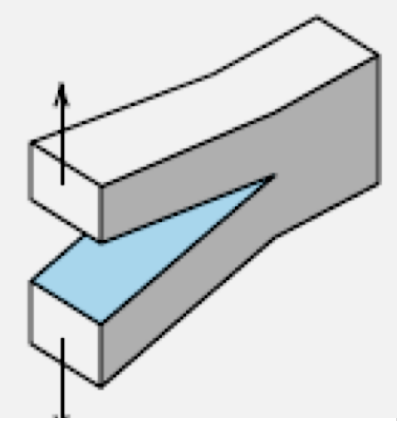
$$Y(a/W) = \left(\frac{W}{\pi a}\tan\frac{\pi a}{W}\right)^{1/2}$$

- Y – dimensionless constant is f(size and shape of flaw, size, shape, and loading mode of sample)
- As a/W approaches zero (very wide sample, very small crack) Y approaches unity

Y = 1



Review of basic fracture mechanics: single value measures of fracture toughness



critical stress intensity, Kc is the stress intensity factor for which unstable crack growth occurs

- Kc is general
- **KIc** is for mode I plane strain
- Y depends on crack and sample geometry

$$K_{Ic} = Y\sigma\sqrt{\pi a}$$

The stress intensity factor, K, and fracture toughness, KIc, are analogous to and applied stress and yield strength.

- You can apply many stress magnitudes to the material, but yield only occurs at one value, the yield strength.
- Similarly, you can apply a stress remotely that produces a K near the crack. Only one K, the critical or KIc, causes fracture.

KIc is sensitive to temperature, strain rate, and microstructure –

- it decreases with increasing strain rate and decreasing temperature.
- Higher strength materials typically have lower KIc
- Features/structures/characteristics that arrest crack growth enhance fracture toughness

Some fracture toughness values for engineering materials

(you need to have sense of the value of fracture toughness for the different material classes: metals, ceramics, and of course, bone).

TABLE 5.1. Fracture toughness for several materials

Material	K_{Ic}, MPa-m$^{1/2}$
2024 Aluminum	20–40
4330V Steel	86–110
Ti-6Al-4V	106–123
Concrete	0.23–1.43
Al_2O_3 ceramic	3–5.3
SiC ceramic	3.4
PMMA polymer	0.8–1.75
Polycarbonate polymer	2.75–3.3
Cortical bone	2.2–6.3

ductile, tough

brittle

Some *modulus* and *fracture toughness* values for materials ***(you should have sense of these values).***

Table 2 Typical range of (mode I) single-value fracture toughness values given in both K and G for cortical bone in comparison to dentin another mineralized tissue and a number of common engineering materials

Material	E (GPa)	K_c (MPa$\sqrt{m}$)
Cortical bone	15–25	2–7
Dentin	20–25	1–2
Titanium alloy (Ti–6Al–4V)	110	65–100
High strength steel (4340)	210	50–100
Commercial silicon carbide	400	3
Silicon	110	1

Fracture toughness for bone – what are C-L, L-C, L-R???

Table 1 Examples of mode-I single-value fracture toughness results for cortical bone using compact tension, C(T), and single-edge-notched bend, SEN(B), specimens taken from various sources

Species	Bone	Orientation[a]	K_c (MPa $\sqrt{m}$)	G_c (J/m^2)	Test geometry	References
Bovine	Femur	Long	3.6 ± 0.7		C(T)	32
Bovine	Femur	Long	2.4–5.2[b]	920–780[b]	C(T)	115
Bovine	Femur	Transverse	5.7 ± 1.4		SEN(B)	133
Bovine	Femur	L-R	3.4–5.1[c]		SEN(B)	20
Bovine	Femur	C-L	2.1–2.9[c]		SEN(B)	20
Bovine	Tibia	Long	4.5–5.4[b]	760–130[b]	C(T)	116
Bovine	Tibia	Long	2.8–6.3[b]	630–880[b]	C(T)	33
Bovine	Tibia	Long	3.2		C(T)	35
Bovine	Tibia	Transverse	6.4		C(T)	35
Bovine	Tibia	L-R	4.5–6.6[c]		SEN(B)	20
Baboon	Femur	Long	1.8 ± 0.5		C(T)	36
Baboon	Femur	Transverse	6.2 ± 0.7		SEN(B)	36
Baboon	Femur	Long	1.7–2.3[d]		C(T)	55
Human	Femur	L-C	6.4 ± 0.3		SEN(B)	24
Human	Femur	C-L		520–190	C(T)	38
Human	Tibia	C-L		400–250	C(T)	38
Human	Tibia	C-L	4.1–4.3[c]	600–30[c]	C(T)	34
Human	Humerus	C-R	2.2 ± 0.2		SEN(B)	37
Human	Humerus	C-L	3.5 ± 0.1		SEN(B)	37
Human	Humerus	L-C	5.3 ± 0.4		SEN(B)	37
Human	Femur	Transverse	4.3–5.4[d]		SEN(B)	19

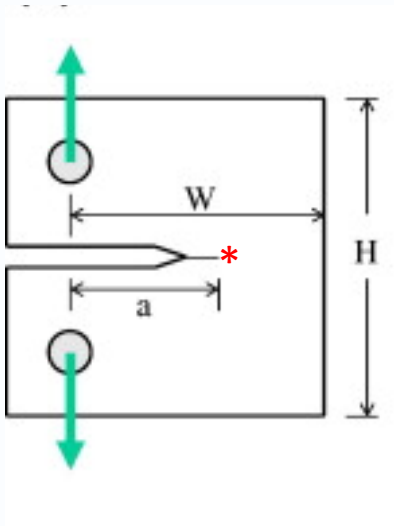
Data are given in either K or G as reported by the authors. All reported values are mean values, standard deviations are given when possible.
[a]When specific orientation is unknown, cracking direction is given, see Fig. 3 for details.
[b]Range of mean values for several sets of data from samples tested at different loading rates.
[c]Range of mean values for two sets of data using samples of different thickness.
[d]Range of mean values for three sets of data using samples from different age groups.
[e]Range of mean values for two sets of data using samples stored in different media.

Don't worry about critical energy release rate

Test geometries from the previous slide, Table 1

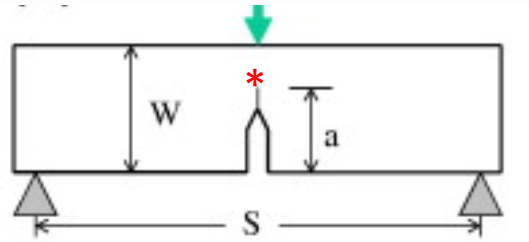
C(T)

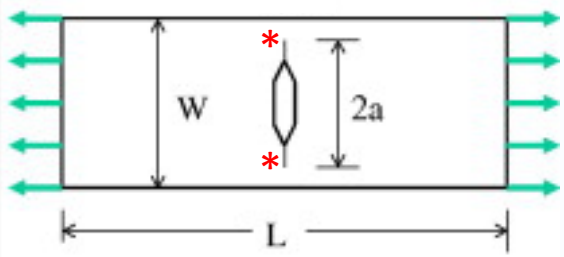
Compact tension



SEN(B)

Single edge-notched bend specimen



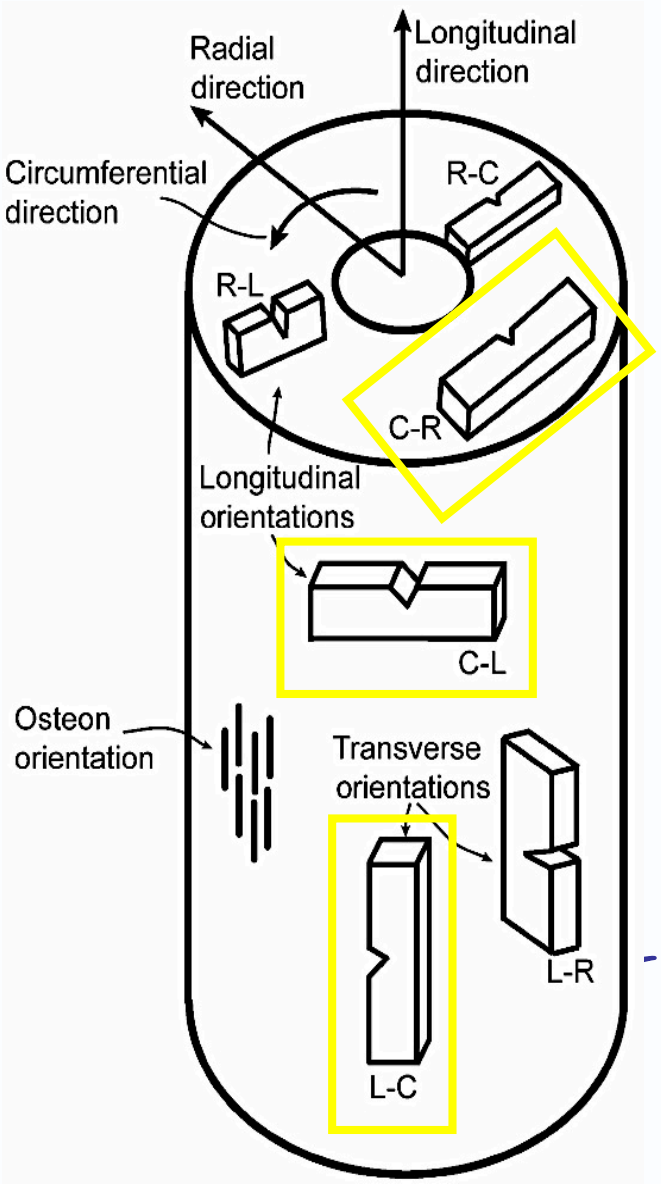


Middle cracked specimen (we see this a lot in fracture mechanics so it is included.)

The very thin grey lines usually indicate that a precrack with known size was purposely generated. See *s

Zhu and Joyce, 2010

Nomenclature for test orientation (Ritchie)



- First letter – normal to crack plane
- Second letter – direction of crack propagation

- L-C, L-R crack runs transverse to the osteons
- C-L, R-L crack runs parallel to osteons (less tough)

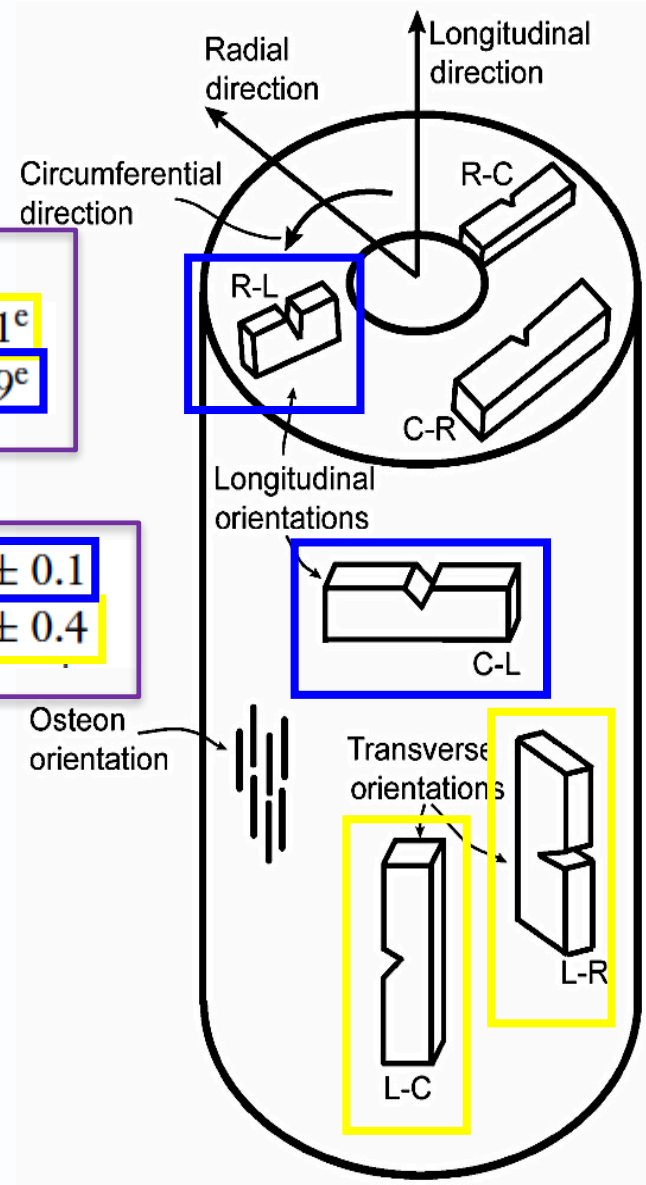
- L-C and C-L easiest to machine, in particular for smaller bones
- R-C, C-R not common in literature because it is difficult to make the samples

Transversely oriented samples are tougher than longitudinal

Why??

Bovine	Femur	L-R	3.4–5.1[e]
Bovine	Femur	C-L	2.1–2.9[e]

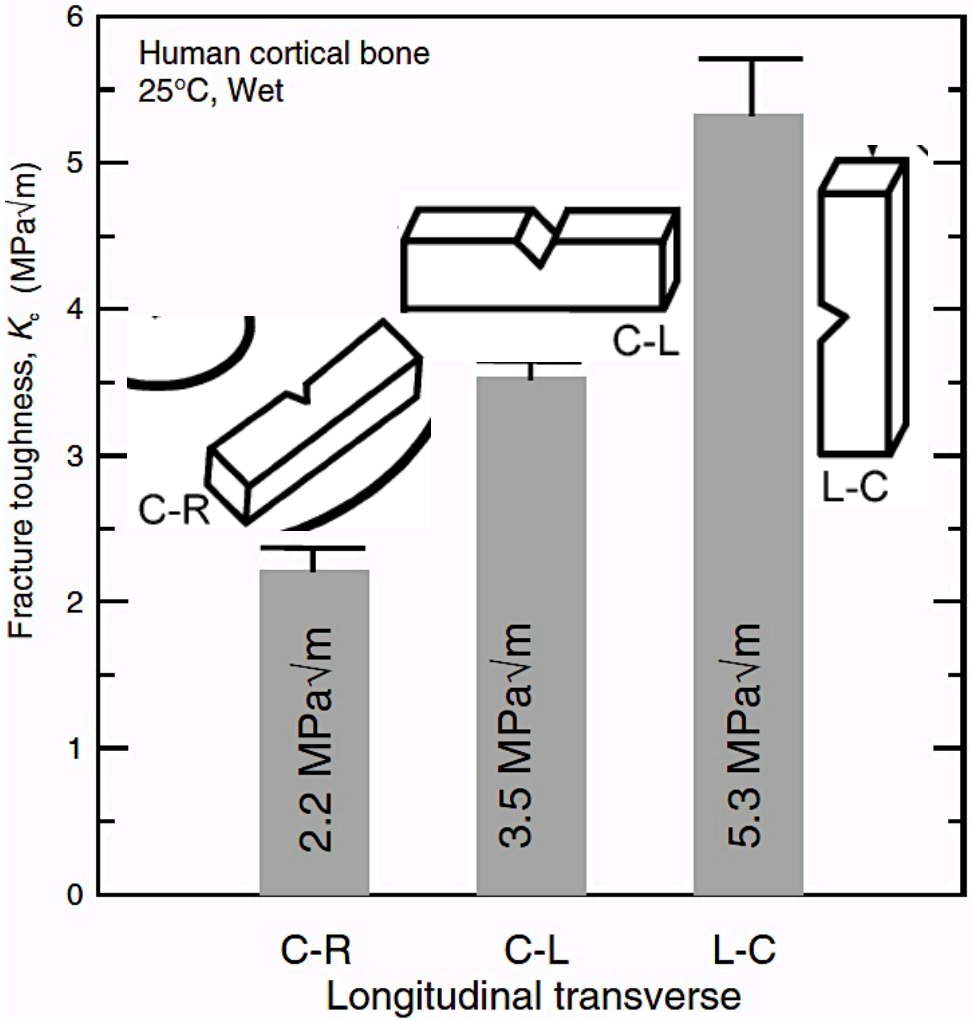
Human	Humerus	C-L	3.5 ± 0.1
Human	Humerus	L-C	5.3 ± 0.4

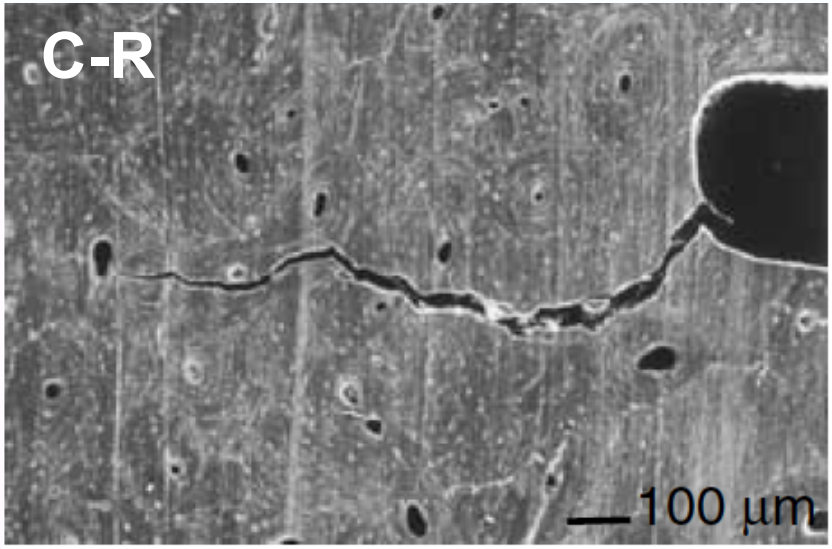


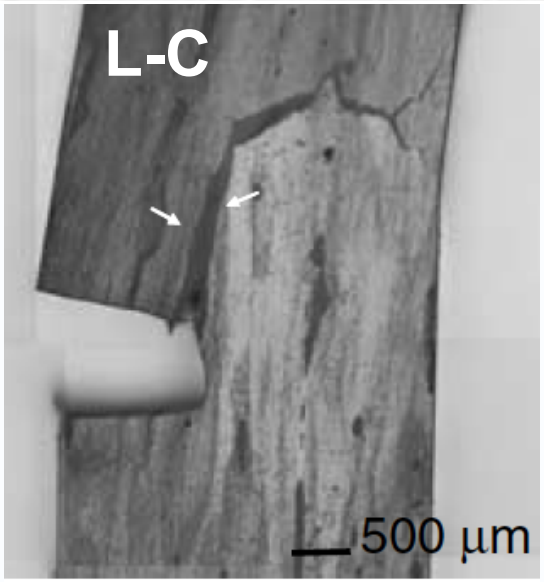
NOTE: Blue box means longitudinal orientation. Yellow means transverse. Kc transverse > Kc longitudinal

Data taken from large table ("Table 1") of cortical bone fracture toughness.

Transversely oriented samples are tougher than longitudinal Propagation relative to osteons….



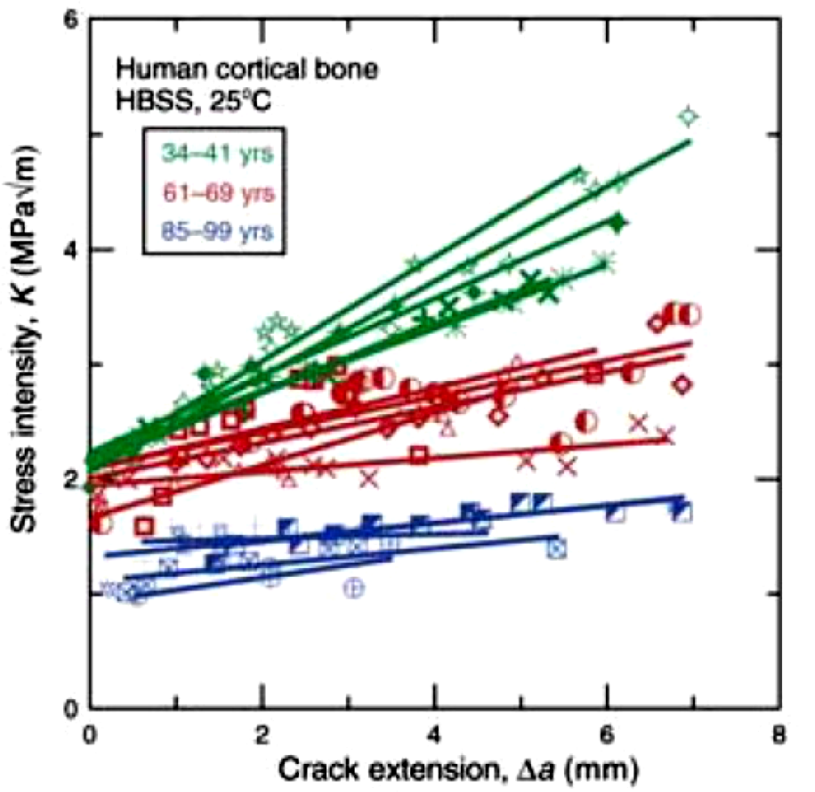




Measures of toughness that are not constant valued: Resistance Curve (R-curve) Behavior

- In some materials fracture resistance (i.e. stress intensity factor) *increases* with crack growth (!!)
- This is caused by *extrinsic toughening mechanisms* (described/defined in next couple of slides)
- Requires an *R-Curve* (resistance-curve) approach to describe the fracture toughness behavior
- Measure instead the toughness or crack extension resistance (i.e. K) vs crack extension, da.
- Begin with a crack initiation toughness, K_0
- Slope of this curve is a measure of the ***crack-growth toughness***
- Can be used to show the presence or operation of extrinsic toughening mechanisms

K increases with crack extension when toughness not single valued
Resistance curve fracture mechanics approach more appropriate



What do we observe from these data?

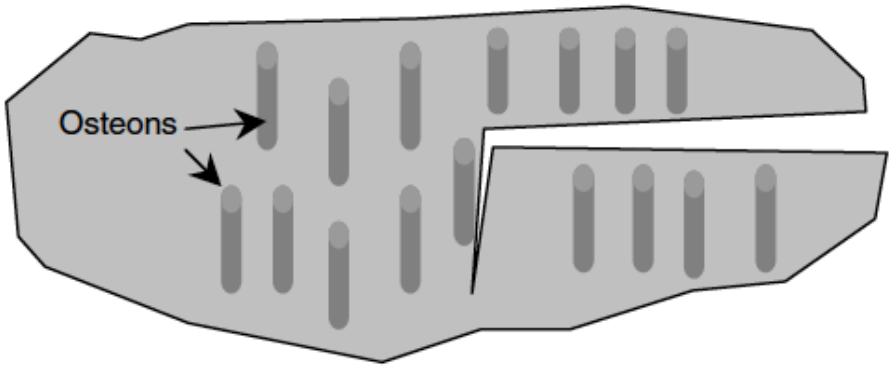
1. Decrease in crack initiation toughness, Ko, with age (intercept)
2. Decrease in crack growth toughness with age (slope)
3. Oldest group is nearly flat (little contribution from extrinsic mechanisms)

What is important about extrinsic vs intrinsic mechanisms?

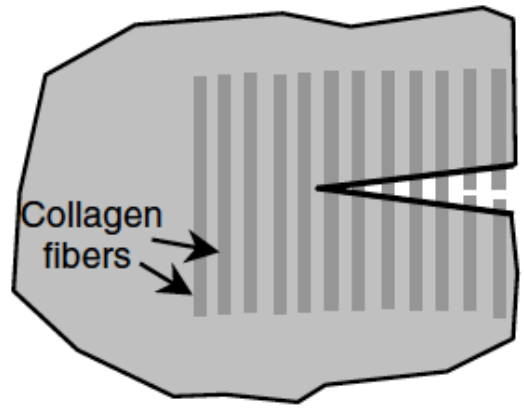
- Crack propagation can be slowed/stopped by both intrinsic and extrinsic mechanisms
  - Intrinsic mechanisms – operate ahead of the crack tip
  - Extrinsic mechanisms – act to shield the crack from the applied stress and act primarily behind the crack tip in the 'crack wake.'
- With extrinsic toughening mechanisms, fracture resistance increases with extension – this typically happens in bone
- They act to reduce the stress intensity factor at the crack tip and 'shield' the crack

Intrinsic: resistance *ahead* of crack
Extrinsic: resistance *behind* crack

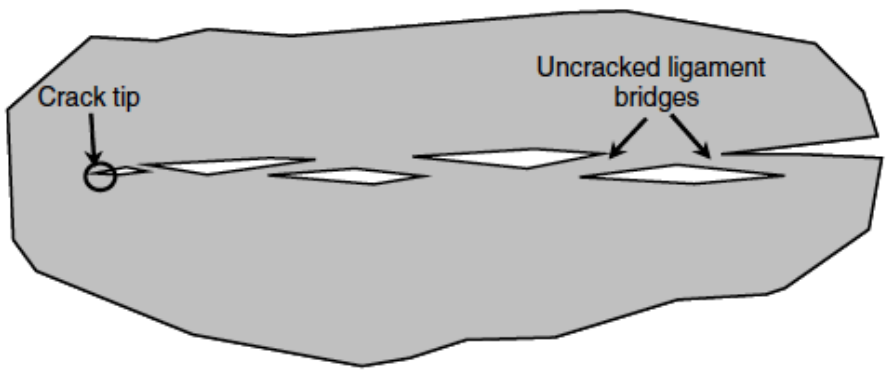
Which are these?
Why?



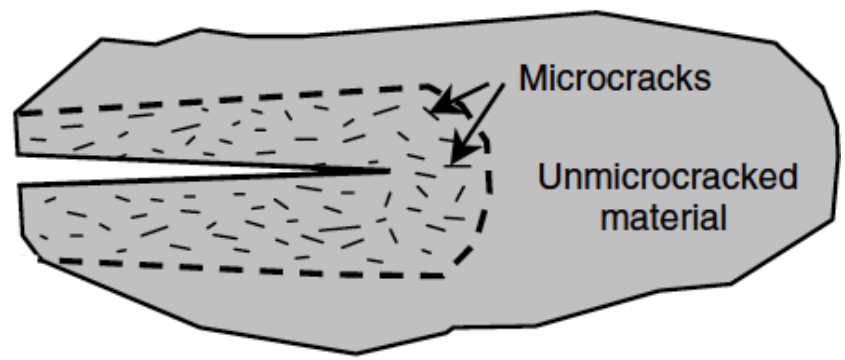
(a) Crack deflection



(b) Crack bridging



(c) Uncracked ligament bridging



(d) Microcracking

Extrinsic mechanisms cause an increase in fracture resistance with crack extension (more info in lecture notes)

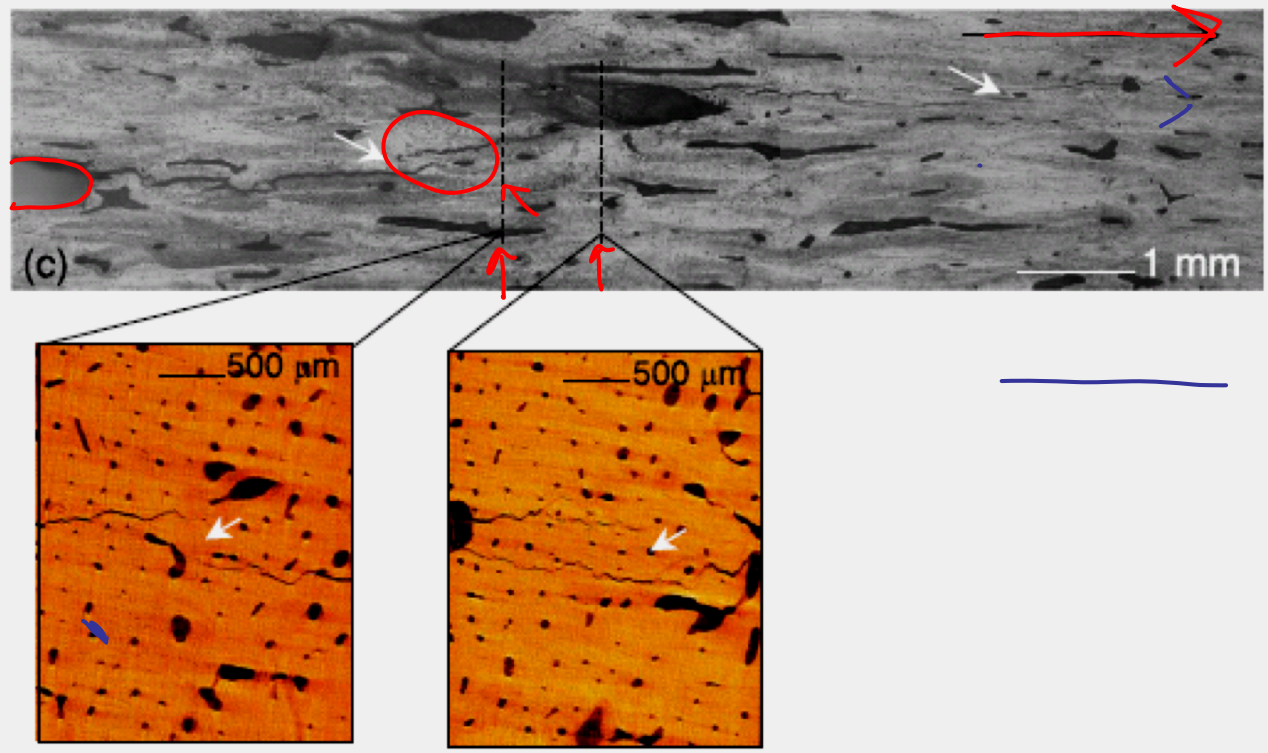
What are these mechanisms? How do they operate?

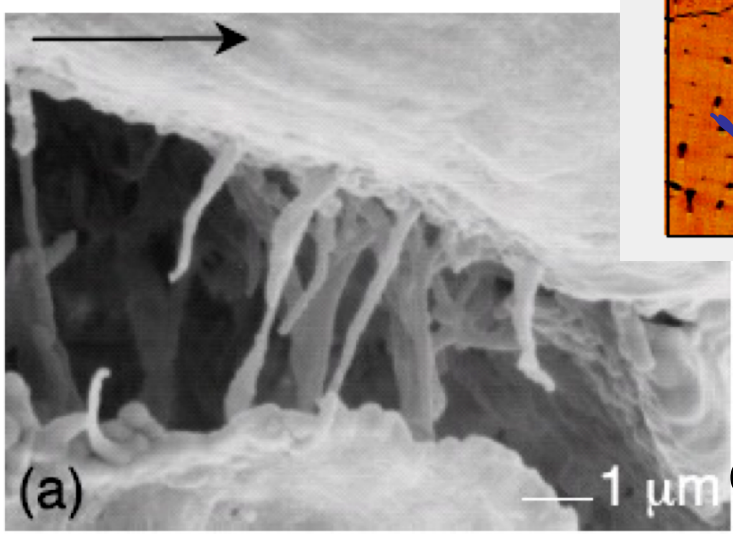
- Intrinsic – interfaces and features that deflect, impede the crack
- What are the extrinsic mechanisms in bone?
- How do extrinsic mechanisms increase resistance?
  - Intact bridges (uncracked ligaments or collagen fibrils) spanning crack sustain part of load in the region, reducing the stress intensity at the tip by and amount called Kbr – the bridging stress intensity (we will estimate for both coll fibers and intact ligament bridges)
  - microcracks
    - Volume of damage zone w/ microcracks increases
    - Constrained by rest of the material that is more rigid
    - Compressive stress at crack tip
    - Modulus is reduced within the zone of microcracks

Microstructural evidence of extrinsic mechanisms

notch

Uncracked ligament bridging

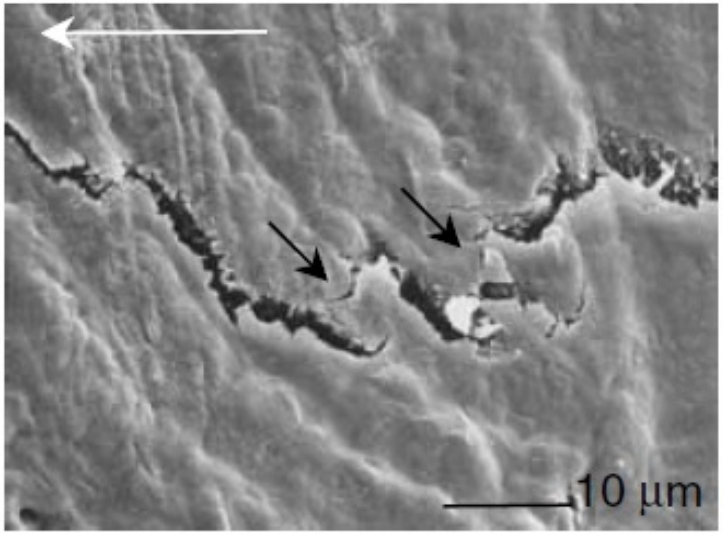




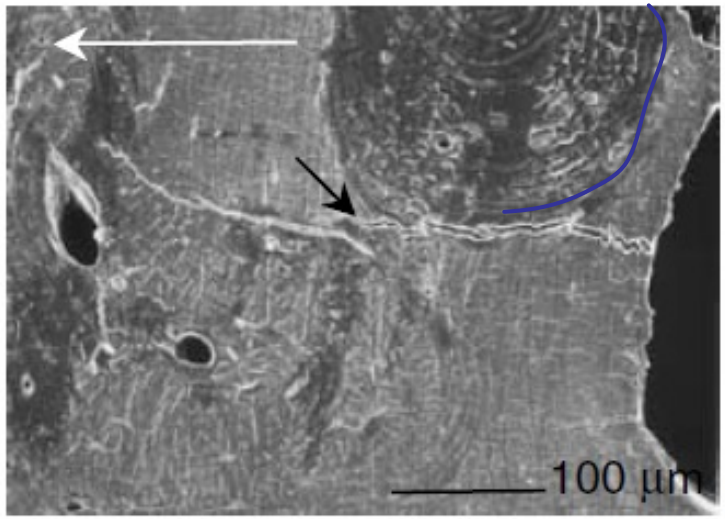
Crack bridging by collagen fibers

Fig. 1. Scanning electron micrographs of (a) crack bridging by collagen fibers in human cortical bone and (b) microcracking (indicated by white arrows) at tubule sites in human dentin. (c) Evidence of uncracked ligament bridging (indicated by white arrows) shown for a crack in a 61-year-old human cortical bone in an optical micrograph (top) and in X-ray computed tomographic reconstructions of through-thickness slices (Nalla RK, Kinney JH, Ritchie RO, unpublished research, 2004). The horizontal black arrow in each case indicates the direction of nominal crack growth.

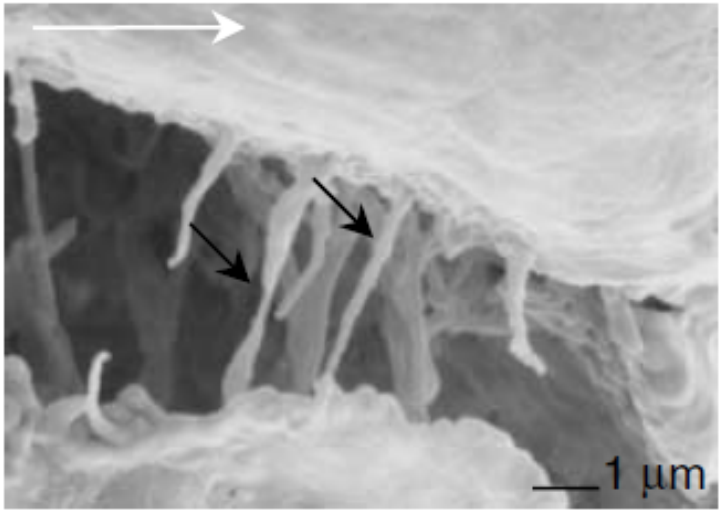
More examples of extrinsic toughening…



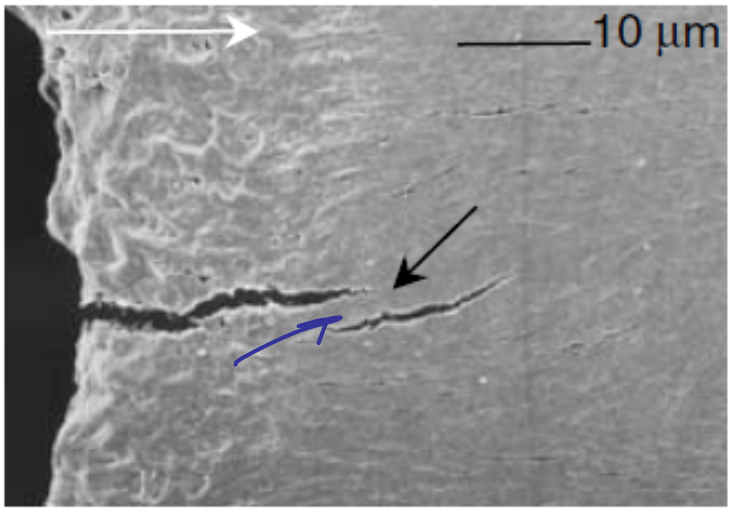
(a)



(b)

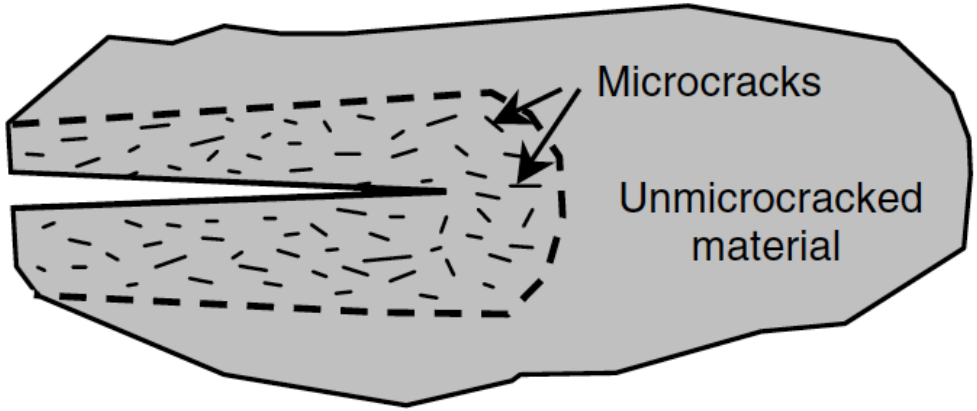


(c)



(d)

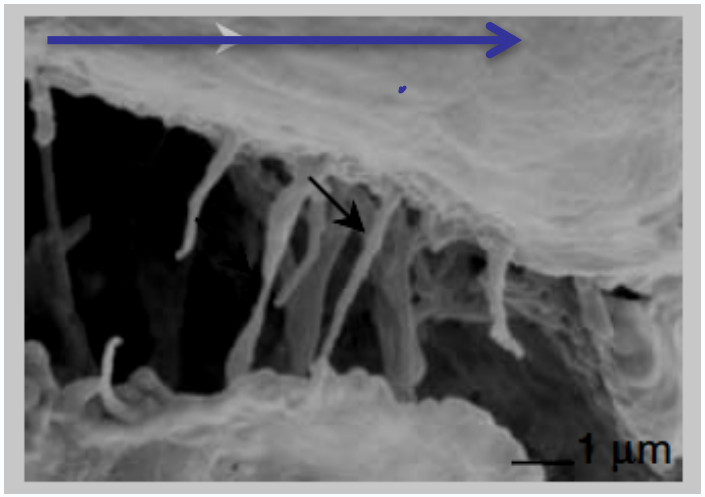
Constrained microcracking: mechanisms and estimated contribution (class notes)

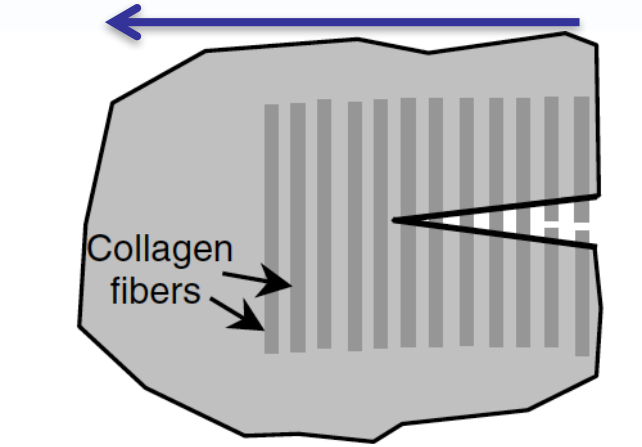


(d) Microcracking

Estimates of toughening by collagen fiber bridging

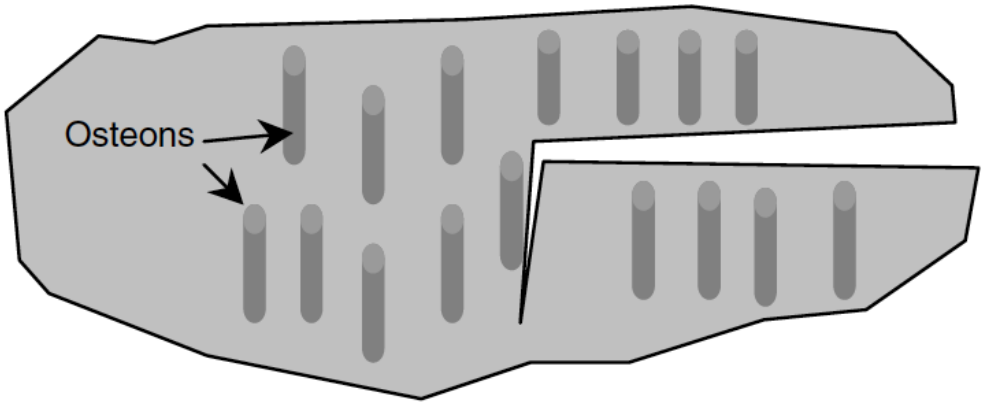
(class notes)



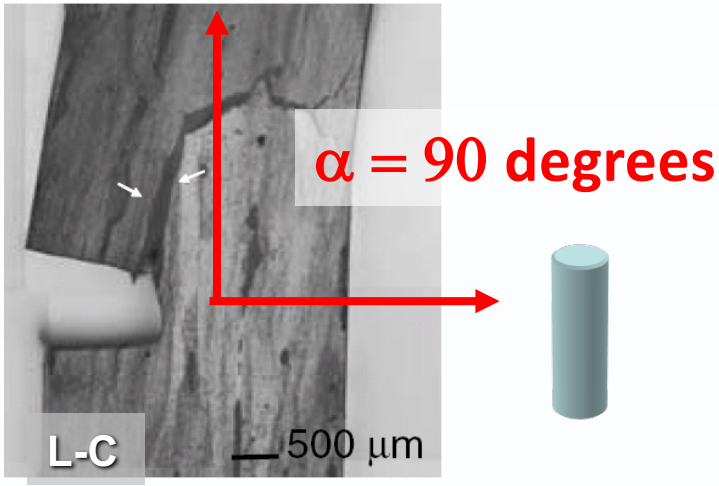


(b) Crack bridging

Simple analysis for estimating stress intensity for L-C (highest) main toughening mechanism crack deflection



(a) Crack deflection

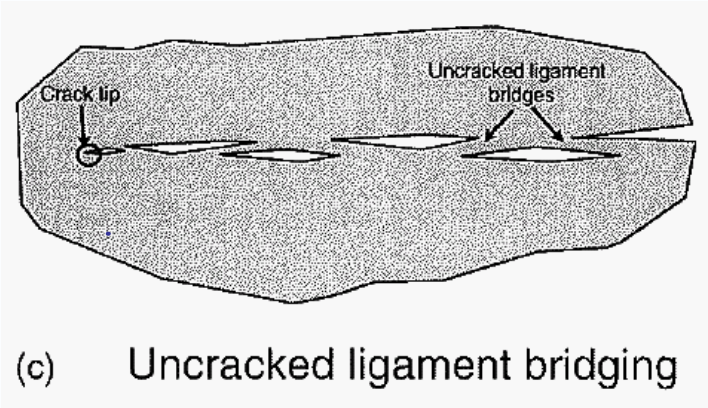


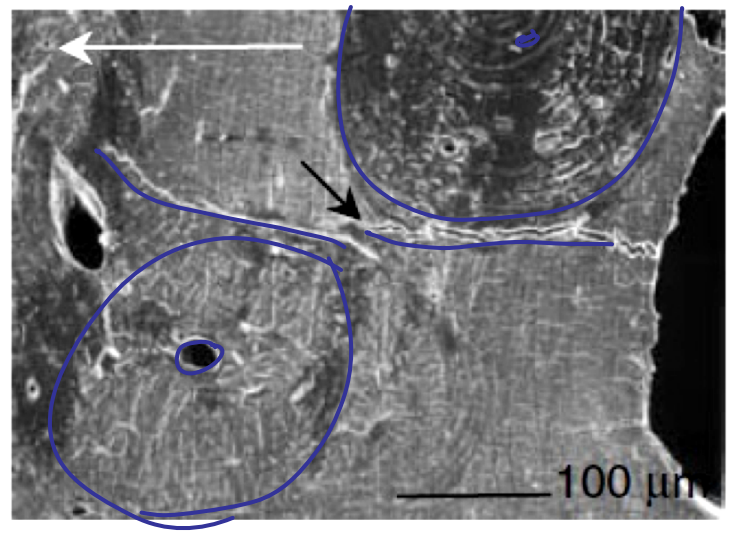
(class notes)

Simple analysis for estimating stress intensity for L-C

(class notes)

Estimates of toughening by bridging ligaments



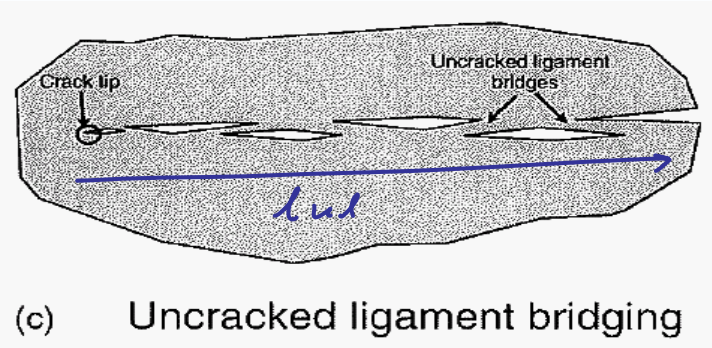


Ex. of bridging for C-R orientation

(class notes)

Estimates of toughening by bridging ligaments

(C-R)



(c) Uncracked ligament bridging

(class notes)

Direction of normal to crack plane
– direction of crack propagation

Be able to describe (using schematics and text)
which of the crack plane direction –
crack propagation direction combinations have higher toughness and why.

Be able to:

- Define and determine K_0 from the plot

- Define and determine crack-growth toughness

- Indicate how age affects the R-curve behavior (e.g. order the R-curves based on age)

take away these points on bone fracture mechanics

- Bone is tougher in some directions compared to others (anisotropic).
  - What orientations are toughest? Least tough?
  - Why are some orientations tougher?
  - What mechanisms make one direction more tough than another?
- Fracture toughness of bone may not be single valued and may need to be described using R-curve approach.
  - What does an R-curve measure?
  - How the measure of toughness with an R-curve different from the material property of fracture toughness, K_{IC}?
  - What specific toughening mechanisms cause toughening to increase with increasing crack growth? And how do they increase the toughness?
  - How are these types of mechanisms classified?
  - Which ones contribute most to each orientation?