MATH 1115

FUNDAMENTAL MATHEMATICS FOR THE GENERAL SCIENCES I

Extra Questions and Solutions in S.I. Units

Question 1:

A desk has an area of 1.04 m^2. Express this area in:

(a) cm^2

(b) μm^2

(c) km^2

Solution:

a)

$1 \text{ m} = 100 \text{ cm, so } 1 \text{ m}^2 = 100^2 \text{ cm}^2$

$\text{Thus } 1.04 \text{ m}^2 = 1.04 \times 100^2 \text{ cm}^2 = 1.04 \times 10^4 \text{ cm}^2$

b)

$1 \text{ m} = 10^6 \text{ } \mu\text{m, so } 1 \text{ m}^2 = \left(10^6\right)^2 \text{ } \mu\text{m}^2$

$\text{Thus } 1.04 \text{ m}^2 = 1.04 \times \left(10^6\right)^2 \text{ } \mu\text{m}^2 = 1.04 \times 10^{12} \text{ } \mu\text{m}^2$

c)

$1 \text{ km} = 10^3 \text{ m, so } 1 \text{ km}^2 = \left(10^3\right)^2 \text{ m}^2$

$\text{Thus } 1 \text{ m}^2 = \frac{1}{(10^3)^2} \text{ km}^2$

$\text{and } 1.04 \text{ m}^2 = \frac{1.04}{(10^3)^2} \text{ km}^2 = 1.04 \times 10^{-6} \text{ km}^2$

Question 2:

Convert 2.7 $\mathrm{g\,cm^{-3}}$ (the density of the specimen of granite shown in Figures 3.1 and 3.2) to a value in the SI units of $\mathrm{kg\,m^{-3}}$.

Answer

$1\ \mathrm{kg} = 10^3\ \mathrm{g}$, so $1\ \mathrm{g} = \frac{1}{10^3}\ \mathrm{kg} = 10^{-3}\ \mathrm{kg}$

$1\ \mathrm{m} = 10^2\ \mathrm{cm}$, so $1\ \mathrm{m}^3 = \left(10^2\right)^3\ \mathrm{cm}^3 = 10^6\ \mathrm{cm}^3$ (from Section 3.4.2)

so $1\ \mathrm{cm}^3 = \frac{1}{10^6}\ \mathrm{m}^3 = 10^{-6}\ \mathrm{m}^3$

To convert from $\mathrm{g\,cm^{-3}}$ to $\mathrm{kg\,m^{-3}}$ we need to *multiply* by 10^{-3} (to convert the g to kg) and *divide* by 10^{-6} (to convert the $\mathrm{cm^{-3}}$ to $\mathrm{m^{-3}}$).

$$1\ \mathrm{g\,cm^{-3}} = \frac{10^{-3}}{10^{-6}}\ \mathrm{kg\,m^{-3}} = 10^{-3-(-6)}\ \mathrm{kg\,m^{-3}} = 10^3\ \mathrm{kg\,m^{-3}}$$

Thus $2.7\ \mathrm{g\,cm^{-3}} = 2.7 \times 10^3\ \mathrm{kg\,m^{-3}}$.

The specimen of granite has a density of $2.7 \times 10^3\ \mathrm{kg\,m^{-3}}$.

Question 3:

The World Health Organization reduced its maximum recommended concentration for arsenic in drinking water from 50 $\mathrm{\mu g\,l^{-1}}$ to 10 $\mathrm{\mu g\,l^{-1}}$ in 1999. Convert 10 $\mathrm{\mu g\,l^{-1}}$ to a value in:

(a) $\mathrm{\mu g\,ml^{-1}}$

(b) $\mathrm{mg\,dm^{-3}}$

(c) $\mathrm{g\,m^{-3}}$

Note: Some textbooks use a common letter " l " to represent litre but the better convention and the one we use is capital "L".

Solution:

a)

$1\ \mathrm{l} = 10^3\ \mathrm{ml}$

To convert from $\mu\mathrm{g\,l^{-1}}$ to $\mu\mathrm{g\,ml^{-1}}$ we need to *divide* by 10^3.

$$1\ \mu\mathrm{g\,l^{-1}} = \frac{1}{10^3}\ \mu\mathrm{g\,ml^{-1}} = 10^{-3}\ \mu\mathrm{g\,ml^{-1}}$$

$$10\ \mu\mathrm{g\,l^{-1}} = 10 \times 10^{-3}\ \mu\mathrm{g\,ml^{-1}}$$
$$= 1.0 \times 10^{-2}\ \mu\mathrm{g\,ml^{-1}} \text{ to two significant figures.}$$

b)

Note that $10\ \mu\mathrm{g\,l^{-1}} = 10\ \mu\mathrm{g\,dm^{-3}}$, since 1 litre is defined to be equal to $1\ \mathrm{dm^3}$ (Section 3.4.2).

$1\ \mathrm{mg} = 10^3\ \mu\mathrm{g}$

so

$$1\ \mu\mathrm{g} = \frac{1}{10^3}\ \mathrm{mg} = 10^{-3}\ \mathrm{mg}$$

To convert from $\mu\mathrm{g\,dm^3}$ to $\mathrm{mg\,dm^3}$ we need to *multiply* by 10^{-3}.

$$1\ \mu\mathrm{g\,dm^3} = 10^{-3}\ \mathrm{mg\,dm^3}$$

$$10\ \mu\mathrm{g\,dm^3} = 10 \times 10^{-3}\ \mathrm{mg\,dm^3}$$
$$= 1.0 \times 10^{-2}\ \mathrm{mg\,dm^3} \text{ to two significant figures.}$$

So a concentration of $10\ \mu\mathrm{g\,l^{-1}}$ is equal to $1.0 \times 10^{-2}\ \mathrm{mg\,dm^3}$.

c)

Note that 10 $\mu g\, l^{-1} = 10\ \mu g\, dm^{-3}$.

$$1\ g = 10^6\ \mu g$$
$$\text{so } 1\ \mu g = \frac{1}{10^6}\ g = 10^{-6}\ g$$

$$1\ m = 10\ dm$$
$$\text{so } 1\ m^3 = 10^3\ dm^3$$
$$\text{and } 1\ dm^3 = \frac{1}{10^3}\ m^3 = 10^{-3}\ m^3$$

To convert from $\mu g\, dm^{-3}$ to $g\, m^{-3}$ we need to *multiply* by 10^{-6} (to convert the μg to g) and *divide* by 10^{-3} (to convert the dm^{-3} to m^{-3}).

$$1\ \mu g\, dm^{-3} = \frac{10^{-6}}{10^{-3}}\ g\, m^{-3}$$

$$\begin{aligned} 10\ \mu g\, dm^{-3} &= 10 \times \frac{10^{-6}}{10^{-3}}\ g\, m^{-3} \\ &= 10 \times 10^{-6-(-3)}\ g\, m^{-3} \\ &= 10 \times 10^{-3}\ g\, m^{-3} \\ &= 1.0 \times 10^{-2}\ g\, m^{-3} \text{ to two significant figures.} \end{aligned}$$

So a concentration of 10 $\mu g\, l^{-1}$ is equal to $1.0 \times 10^{-2}\ g\, m^{-3}$.