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MATH 1115 - FUNDAMENTAL MATHEMATICS FOR GENERAL SCIENCES I (GROUP 1)
ASSIGNMENT 2 SOLUTIONS

1) A

Since $\left(\frac{25}{4}\right)^{-1/2} = \frac{25^{-1/2}}{4^{-1/2}} = \frac{(5^2)^{-1/2}}{(2^2)^{-1/2}} = \frac{5^{-1}}{2^{-1}} = \frac{\frac{1}{5}}{\frac{1}{2}} = \frac{1}{5} \times \frac{2}{1} = \frac{2}{5}$

2) B

$\sqrt{\frac{3}{2}} = \frac{\sqrt{3}}{\sqrt{2}}$ and $\sqrt{3}$ and $\sqrt{2}$ are both surds.

3) a) $\left(\frac{-5}{6} \div \left(\frac{216}{25}\right)^{1/3}\right) \times \left(\frac{5}{6}\right)^{-1}$

$= \left(\frac{-5}{6} \div \frac{(6^3)^{1/3}}{(5^2)^{1/3}}\right) \times \left(\frac{6}{5}\right)$

$= \left(\frac{-5}{6} \div \frac{6}{5^{2/3}}\right) \times \frac{6}{5}$

$= \left(\frac{-5}{6} \times \frac{5^{2/3}}{6}\right) \times \frac{6}{5}$

$= \frac{-5^{5/3}}{\cancel{36}\,6} \times \frac{\cancel{6}\,1}{5}$

$= \frac{-5^{5/3 - 1}}{6}$

$= \frac{-5^{2/3}}{6}$

(Answer can be left in this form).

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3b) $\left(-\frac{22}{9} \times \left(\frac{9}{4}\right)^{3/2}\right) \div \frac{11}{2}$

$= \left(-\frac{22}{9} \times \frac{(3^2)^{3/2}}{(4^2)^{3/2}}\right) \div \frac{11}{2}$

$= \left(-\frac{22}{9} \times \frac{3^3}{4^3}\right) \div \frac{11}{2}$

$= \left(\frac{-22}{\cancel{3^2}} \times \frac{\cancel{3^3}\,1}{64}\right) \div \frac{11}{2}$

$= \left(-\overset{2}{\cancel{22}} \times \frac{3}{64}\right) \times \frac{2}{\cancel{11}}$

$= -\frac{12}{64}$

$= -\frac{3}{16}$

4) a) $9^{(1-2x)} = 27^{-\left(\frac{x}{3}\right)}$

$\Rightarrow (3^2)^{(1-2x)} = (3^3)^{-\left(\frac{x}{3}\right)}$

$\Rightarrow 3^{2(1-2x)} = 3^{3\left(-\frac{x}{3}\right)}$

$\Rightarrow 3^{2-4x} = 3^{-x}$

Equating powers (since we have the same base)

$2-4x = -x$

$2 = -x+4x$

$2 = 3x$

Hence $x = \frac{2}{3}$

4 b) $$\frac{2^{3x+7}}{4^{2x-2}} = \frac{8^{x-3}}{32^{5-x}}$$

$$\Rightarrow \frac{2^{3x+7}}{(2^2)^{(2x-2)}} = \frac{(2^3)^{(x-3)}}{(2^5)^{(5-x)}}$$

$$\frac{2^{3x+7}}{2^{4x-4}} = \frac{2^{3x-9}}{2^{25-5x}}.$$

$$2^{(3x+7)-(4x-4)} = 2^{(3x-9)-(25-5x)}$$

$$\Rightarrow 2^{3x+7-4x+4} = 2^{3x-9-25+5x}$$

$$2^{-x+11} = 2^{8x-34}$$

Equating powers gives.

$$-x + 11 = 8x - 34$$

$$11 + 34 = 8x + x$$

$$45 = 9x$$

$$\frac{45}{9} = x$$

Hence $x = 5$

5) a) $$\frac{5+\sqrt{6}}{\sqrt{3}-\sqrt{2}} = \frac{(5+\sqrt{6})(\sqrt{3}+\sqrt{2})}{(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})}$$

$$= \frac{5\sqrt{3} + 5\sqrt{2} + (\sqrt{6})(\sqrt{3}) + (\sqrt{6})(\sqrt{2})}{(\sqrt{3})^2 - (\sqrt{2})^2}$$

$$= \frac{5\sqrt{3} + 5\sqrt{2} + \sqrt{18} + \sqrt{12}}{3-2}.$$

(4)

$$= \frac{5\sqrt{3}+5\sqrt{2}+\sqrt{9\times2}+\sqrt{4\times3}}{1}$$

$$= \frac{5\sqrt{3}+5\sqrt{2}+3\sqrt{2}+2\sqrt{3}}{1}$$

$$= 7\sqrt{3}+8\sqrt{2}$$

b) $$\frac{1}{2+\sqrt{2}} = \frac{1}{2+\sqrt{2}}\frac{(2-\sqrt{2})}{(2-\sqrt{2})}$$

$$= \frac{2-\sqrt{2}}{2^2-(\sqrt{2})^2}$$

$$= \frac{2-\sqrt{2}}{4-2}$$

$$= \frac{2-\sqrt{2}}{2}$$

Note: There are a number of methods for S.I. units

6) $10\,\mu g\,L^{-1} \rightarrow \mu g\,mL^{-1}$

Now $1\,L = 10^3\,mL$

$10\,\mu g\,L^{-1}$ means in $1\,L \rightarrow 10\,\mu g$

$\Rightarrow 10^3\,mL \rightarrow 10\,\mu g$

$\Rightarrow 1\,mL = \frac{10\,\mu g}{10^3}$

$= 10^{-2}\,\mu g$

$\therefore$ $10\,\mu g\,L^{-1} \equiv 1.0 \times 10^{-2}\,\mu g\,mL^{-1}$

(Note: Students can also apply laws of indices to obtain correct answer, i.e.,

Alternative

$$1\,L = 10^{3}\,mL$$
$$(1L)^{-1} = (10^{3}\,mL)^{-1}$$
$$1\,L^{-1} = 10^{-3}\,mL^{-1}$$
$$\therefore\ 10\,\mu g\,L^{-1} = 10 \times 10^{-3}\,\mu g\,L^{-1}$$
$$= 1.0 \times 10^{-2}\,\mu g\,L^{-1}$$

6b) $10\,\mu g\,L^{-1} \rightarrow mg\,cm^{-3}$

This question can also be approached in more than one methods.

Consider $\mu g\,L^{-1} \rightarrow mg\,cm^{-3}$

we can go from $\mu g\,L^{-1} \rightarrow mg\,L^{-1}$

then $mg\,L^{-1} \rightarrow mg\,cm^{-3}$

Now $10\mu g L^{-1}$ means in 1 L we have $10\,\mu g$.

But $1\,\mu g = 10^{-6}\,g \Rightarrow 10^{6}\,\mu g = 1\,g$ — (1)

and $1\,mg = 10^{-3}\,g \Rightarrow 10^{3}\,mg = 1\,g$ — (2)

(1) = (2) $\Rightarrow$ $10^{6}\,\mu g = 10^{3}\,mg$

(6)

$$1\ \mu g = \frac{10^3}{10^6} mg$$

$$= 10^{3-6} mg$$

$$= 10^{-3} mg$$

$\therefore\ 10\ \mu g = 10 \times 10^{-3} mg = 10^{-2} mg$

$\therefore$ In 1 litre we have 10^{-2} mg

$$\therefore\quad 10\mu g L^{-1} \equiv 10^{-2} mg L^{-1}$$

Now we need $10^{-2} mg L^{-1} \rightarrow mg\, cm^{-3}$

Using hint : If 1 Litre $= 10^3$ mL

But $1\ mL = 1\ cm^3$

$$\Rightarrow 1\ L \equiv 10^3 cm^3$$

$$\Rightarrow (1L)^{-1} = (10^3 cm^3)^{-1}$$

i.e. $1 L^{-1} = 10^{-3} cm^{-3}$

$$\therefore\ 10^{-2} mg L^{-1} = (10^{-2} \times 10^{-3}) mg\, cm^{-3}$$

$$= 10^{-5} mg\, cm^{-3}$$

$$= 1.0 \times 10^{-5} mg\, cm^{-3}$$

Note: We can also convert in the following way :

First $\mu g L^{-1} \rightarrow \mu g\, cm^{-3}$

then $\mu g\, cm^{-3} \rightarrow mg\, cm^{-3}$

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7) $10.49\,g\,cm^{-3} \rightarrow kg\,m^{-3}$

We use the following method:

$$g\,cm^{-3} \rightarrow kg\,cm^{-3}$$

$$kg\,cm^{-3} \rightarrow kg\,m^{-3}$$

Now $1\,g = 10^{-3}\,kg$

$$\therefore 10.49\,g = 10.49 \times 10^{-3}\,kg$$

$$= 1.049 \times 10 \times 10^{-3}\,kg$$

$$= 1.049 \times 10^{-2}\,kg$$

$$\Rightarrow \quad 10.49\,g\,cm^{-3} = 1.049 \times 10^{-2}\,kg\,cm^{-3}$$

Now $1\,cm = 10^{-2}\,m$

$$\Rightarrow 1\,cm^{-3} = (10^{-2}\,m)^{-3}$$

$$= (10^{-2})^{-3}\,m^{-3}$$

$$1\,cm^{-3} = 10^{6}\,m^{-3}$$

So $1.049 \times 10^{-2}\,kg\,cm^{-3} = 1.49 \times 10^{-2} \times 10^{6}\,kg\,m^{-3}$

$$= 1.049 \times 10^{4}\,kg\,m^{-3}$$

8 a) $350\ cm^3 \rightarrow m^3$.

Now $1\ cm = 10^{-2}\ m$

$\Rightarrow 1\ cm^3 = (10^{-2})^3\ m^3$

$= 10^{-6}\ m^3$

$350\ cm^3 = 350 \times 10^{-6}\ m^3$

$= 3.50 \times 10^{2} \times 10^{-6}\ m^3$.

$= 3.50 \times 10^{-4}\ m^3$.

b) $25 \times 10^{-6}\ \mu L \rightarrow m^3$

$1\ \mu L = 10^{-6}\ L \Rightarrow 10^{6}\ \mu L = 1\ L$ — (1)

Now $1\ L = 10^{3}\ mL$

But $1\ mL = 1\ cm^3$ (Using that)

$\Rightarrow 1\ L = 10^{3}\ cm^3$ — (2)

From (1) and (2): $10^{6}\ \mu L = 10^{3}\ cm^3$ — (3)

Now $1\ cm^3 = 10^{-6}\ m^3$ (from 8(a))

$\Rightarrow 10^{3}\ cm^3 = 10^{-6} \times 10^{3}\ m^3$

$= 10^{-3}\ m^3$ — (4)

Using (3) and (4): $10^{6}\ \mu L = 10^{-3}\ m^3$

$\therefore 1\ \mu L = \frac{10^{-3}}{10^{6}}\ m^3 = 10^{-3-6}\ m^3$

$\therefore \; 1\,\mu L = 10^{-9} m^3$

$\therefore \; 25 \times 10^{-6}\,\mu L = 25 \times 10^{-6} \times 10^{-9} m^3$

$= 25 \times 10^{-15} m^3$

$= 2.5 \times 10 \times 10^{-15} m^3$

$= 2.5 \times 10^{-14} m^3$

(ii) $12.2 \times 10^{7} mm^3 \rightarrow m^3$

$1\,mm = 10^{-3} m$

$1\,mm^3 = (10^{-3})^3 m^3 = 10^{-9} m^3$

$12.2 \times 10^{7} mm^3 = 12.2 \times 10^{7} \times 10^{-9} m^3$

$= 12.2 \times 10^{-2} m^3$

$= 1.22 \times 10^{-1} m^3$