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SEM 1 - 2016/2017

MATH 1115 - FUNDAMENTAL MATHEMATICS FOR THE GENERAL SCIENCES I. (GROUP I)

ASSIGNMENT #03 SOLUTIONS

1) $f(x) = 2x-1$, $g(x) = -x$

a) $fg(x) = f[g(x)]$
$= 2(g(x))-1$
$= 2(-x)-1$
$= -2x-1$

b) $gf(x) = g[f(x)]$
$= -(f(x))$
$= -(2x-1)$
$= -2x+1$

2) $f(x) = x+2$, $g(x) = x^2$

a) $fg(x) = f[g(x)]$
$= (g(x))+2$
$= x^2+2$

b) $gf(x) = g[f(x)]$
$= (f(x))^2$
$= (x+2)^2$

3) $f(x) = 3x-1$, $g(x) = x^2+2x-1$

a) $fg(x) = f[g(x)]$
$= 3(g(x))-1$
$= 3(x^2+2x-1)-1$
$= 3x^2+6x-3-1$
$= 3x^2+6x-4$

b) $gf(x) = g[f(x)]$
$= (f(x))^2+2(f(x))-1$
$= (3x-1)^2+2(3x-1)-1$
$= 9x^2 \cancel{-6x}+1+\cancel{6x}-2-1$
$= 9x^2-2$

4) $f(x) = \frac{1}{2x}$, $g(x) = 2x+3$

a) $fg(x) = f[g(x)]$

$= \frac{1}{2[g(x)]}$

$= \frac{1}{2(2x+3)}$

$fg(x) = \frac{1}{4x+6}$

b) $gf(x) = g[f(x)]$

$= 2(f(x))+3$

$= 2\left(\frac{1}{2x}\right)+3$

$gf(x) = \frac{1}{x}+3$

Domain of $fg(x)$ and $gf(x)$

Notice : These functions contain fractions. We cannot have zero in the denominator (or the function is undefined).

$\therefore$ For $fg(x)$: If $4x+6=0$

then $4x=-6$

$x = \frac{-6}{4} = \frac{-3}{2}$

So we cannot have a value of $x = -\frac{3}{2}$

$\therefore$ Domain of $fg(x)$ is $x \in \mathbb{R}$ where $x \neq -\frac{3}{2}$

$\therefore$ Similarly for $gf(x)$: If $x=0$, $\frac{1}{0}$ is undefined

$\therefore$ Domain of $gf(x)$ is $x \in \mathbb{R}$ where $x \neq 0$.

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5) $f(x) = \frac{1}{x-1}$, $g(x) = \frac{1}{x}$

a) $fg(x) = f[g(x)]$

$= \frac{1}{g(x)-1}$

$= \frac{1}{\frac{1}{x} - 1}$

$= \frac{1}{\frac{1-x}{x}}$

$fg(x) = \frac{x}{1-x}$

b) $gf(x) = g[f(x)]$

$= \frac{1}{\cancel{f(x)}}$

$= \frac{1}{\frac{1}{x-1}}$

$gf(x) = x-1$

Domain of composite function $fg(x)$ and $gf(x)$.

For $fg(x)$: If $1-x=0$

$x=1$

Therefore, $x \neq 1$ (so denominator of $fg(x)$ would not be zero).

Domain of $fg(x)$ is $x \neq 1$, $x \in \mathbb{R}$.

Domain of $gf(x)$ is $x \in \mathbb{R}$ (since no restrictions on the functions were specified).

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6) $f(x) = 2x - 3$; $g(x) = (x+1)^2$

a) $f(-3) = 2(-3) - 3 = -6 - 3 = -9$

b) $g(-4) = (-4+1)^2 = (-3)^2 = 9$

c) $fg(x) = f[g(x)]$
$= 2(g(x)) - 3$
$= 2(x+1)^2 - 3$
$= 2(x^2 + 2x + 1) - 3$
$= 2x^2 + 4x + 2 - 3$
$= 2x^2 + 4x - 1$

d) $gf(x) = g[f(x)]$
$= (f(x) + 1)^2$
$= ((2x-3)+1)^2$
$= (2x-2)^2$
OR $4x^2 - 8x + 4$

e) $f^{-1}(x)$:
let $y = 2x - 3$
$x = 2y - 3$
$\frac{x+3}{2} = y$
$\therefore f^{-1}(x) = \frac{x+3}{2}$

$f^{-1}(15)$
$= \frac{15+3}{2}$
$= \frac{18}{2} = 9$

f) $gf^{-1}(x)$:
$\Rightarrow g[f^{-1}(x)]$
$= ((f^{-1}(x)) + 1)^2$
$= \left(\left(\frac{x+3}{2}\right) + 1\right)^2$
$= \left(\frac{x+3+2}{2}\right)^2$
$= \left(\frac{x+5}{2}\right)^2$

$gf^{-1}(15)$
$= \left(\frac{15+5}{2}\right)^2$
$= \left(\frac{20}{2}\right)^2$
$= 10^2$
$= 100$

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7) $f(x) = \frac{2x-1}{2}$, $g(x) = 1-2x$, $h(x) = 2x^2-1$

$fgh(x) = fg[h(x)]$

Now $fg(x) = f[g(x)]$

$= \frac{2(g(x))-1}{2}$

$= \frac{2(1-2x)-1}{2}$

$= \frac{2-4x-1}{2}$

$= \frac{2-4x-1}{2}$

$= \frac{1-4x}{2}$

$\therefore fgh(x) = fg[h(x)]$

$= (1-4(h(x))) \div 2 = \frac{1-4(2x^2-1)}{2}$

$= \frac{1-8x^2+4}{2} = \frac{5-8x^2}{2}$

OR Students can find $gh(x)$ and substitute into f

i.e. $fgh(x) = f[gh(x)]$.

8) Now $g^{-1}(x)$ is the inverse function of $g(x)$

Let $y = 1-2x$

$\Rightarrow x = 1-2y$

$\frac{x-1}{-2} = y$

OR $y = \frac{x-1}{-2}$ $\left(\text{OR } y = \frac{1-x}{2}\right)$

$\therefore\ g^{-1}(x) = \dfrac{x-1}{-2}.$

Also to find $f^{-1}(x)$:

Let $y = \dfrac{2x-1}{2}$

$\Rightarrow x = \dfrac{2y-1}{2}$

$2x = 2y - 1$

$2x + 1 = 2y$

$\dfrac{2x+1}{2} = y$

$\therefore\ f^{-1}(x) = \dfrac{2x+1}{2}.$

Now if $fg(x) = \dfrac{1-\cancel{4}x}{2}$, to find $(fg)^{-1}(x)$

Let $y = \dfrac{1-\cancel{4}x}{2}$

$\Rightarrow x = \dfrac{1-\cancel{4}y}{2}$

$\Rightarrow 2x = 1 - 4y \Rightarrow \dfrac{2x-1}{-4} = y$

$\therefore\ (fg)^{-1}(x) = \dfrac{2x-1}{-4}$ or $\dfrac{1-2x}{4}$

Now $g^{-1}f^{-1}(x) = g^{-1}\left[f^{-1}(x)\right]$

$= \dfrac{\left(f^{-1}(x)\right) - 1}{-2}$

$= \dfrac{\left(\dfrac{2x+1}{2}\right) - 1}{-2}$

$= \dfrac{\dfrac{2x+1-2}{2}}{-2} = \dfrac{(2x-1)}{2} \times -\dfrac{1}{2}$

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$$= -\frac{(2x-1)}{4} \quad \text{or} \quad \frac{1-2x}{4}$$

$$\therefore \ (fg)^{-1}(x) = g^{-1}f^{-1}(x)$$

R.T.S. $(gf)^{-1}(x) = f^{-1}g^{-1}(x)$

Now $gf(x) = g[f(x)]$

$$= 1 - 2(f(x))$$

$$= 1 - 2\left(\frac{2x-1}{2}\right)$$

$$= 1-(2x-1)$$

$$= 1-2x+1$$

$$gf(x) = 2-2x.$$

For $(gf)^{-1}(x)$ (inverse of $gf(x)$)

Let $y = 2-2x$

$\Rightarrow x = 2-2y$

$\frac{x-2}{-2} = y$

$\Rightarrow (gf)^{-1}(x) = \frac{x-2}{-2}$

Now $f^{-1}g^{-1}(x) = f^{-1}[g^{-1}(x)]$

$$= \frac{2(g^{-1}(x))+1}{2}$$

$$= \frac{2\left(\frac{(x-1)}{-2}+1\right)}{2}$$

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$$= \frac{-(x-1) + 2}{2}$$

$$= \frac{-x+1+2}{2}$$

$$\therefore f^{-1}g^{-1}(x) = \frac{-x+2}{2}.$$

Note: this is the same as $\frac{x-2}{-2}$ since $\frac{-x+2}{2} = \frac{-(x-2)}{2}$ or $\frac{(x-2)}{-2}$

$$\therefore (gf)^{-1}(x) = f^{-1}g^{-1}(x).$$

9) $$\frac{1-\sqrt{3}}{2+\sqrt{3}} = \frac{(1-\sqrt{3})}{2+\sqrt{3}} \frac{(2-\sqrt{3})}{(2-\sqrt{3})}$$

$$= \frac{2-\sqrt{3}-2\sqrt{3}+3}{(2)^2-(\sqrt{3})^2}$$

$$= \frac{5-5\sqrt{3}}{4-3}$$

$= 5-5\sqrt{3}$ where $x=5$, $y=5$ in form $x+y\sqrt{3}$.

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10) a) R.T.S.

$$\frac{\sqrt{7}-\sqrt{2}}{\sqrt{7}+\sqrt{2}} = \frac{1}{5}\left(9-2\sqrt{14}\right)$$

Taking L.H.S:

$$\frac{\sqrt{7}-\sqrt{2}}{\sqrt{7}+\sqrt{2}} = \frac{\sqrt{7}-\sqrt{2}}{\sqrt{7}+\sqrt{2}} \cdot \frac{(\sqrt{7}-\sqrt{2})}{\sqrt{7}-\sqrt{2})}$$

$$= \frac{(\sqrt{7})^2-(\sqrt{7})(\sqrt{2})-(\sqrt{2})(\sqrt{7})+(\sqrt{2})^2}{(\sqrt{7})^2-(\sqrt{2})^2}$$

$$= \frac{7-\sqrt{14}-\sqrt{14}+2}{7-2}$$

$$= \frac{9-2\sqrt{14}}{5}$$

$$= \frac{1}{5}\left(9-2\sqrt{14}\right)$$ SHOWN

b) R.T.S. $\frac{\sqrt{7}-\sqrt{2}}{\sqrt{7}+\sqrt{2}} - \frac{\sqrt{7}+\sqrt{2}}{\sqrt{7}-\sqrt{2}} = \frac{-4\sqrt{14}}{5}$.

Solution: Taking L.H.S.

$$\frac{\sqrt{7}-\sqrt{2}}{\sqrt{7}+\sqrt{2}} - \frac{\sqrt{7}+\sqrt{2}}{\sqrt{7}-\sqrt{2}}$$

Note: Simplest way to do this is to rationalise the second fraction (since the first fraction was rationalized in 10(a)).

So rationalizing $\frac{\sqrt{7}+\sqrt{2}}{\sqrt{7}-\sqrt{2}}$

$$= \frac{(\sqrt{7}+\sqrt{2})(\sqrt{7}+\sqrt{2})}{(\sqrt{7}-\sqrt{2})(\sqrt{7}-\sqrt{2})}$$

$$= \frac{(\sqrt{7})^2 + (\sqrt{7})(\sqrt{2}) + (\sqrt{2})(\sqrt{7}) + (\sqrt{2})^2}{(\sqrt{7})^2 - (\sqrt{2})^2}$$

$$= \frac{7 + 2\sqrt{14} + 2}{7-2}$$

$$= \frac{9 + 2\sqrt{14}}{5}$$

$$\therefore \frac{\sqrt{7}-\sqrt{2}}{\sqrt{7}+\sqrt{2}} - \frac{\sqrt{7}+\sqrt{2}}{\sqrt{7}-\sqrt{2}}$$

$$= \frac{1}{5}(9-2\sqrt{14}) - \frac{1}{5}(9+2\sqrt{14})$$

$$= \cancel{\frac{9}{5}} \cancel{-} \cancel{\frac{9}{5}} - \frac{2\sqrt{14}}{5} - \frac{2\sqrt{14}}{5}$$

$$= \frac{-4\sqrt{14}}{5}$$

SHOWN.

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11) a) $\sqrt{28} + \sqrt{252}$

$= \sqrt{7 \times 4} + \sqrt{7 \times 36}$

$= (\sqrt{7} \times 2) + (\sqrt{7} \times 6)$

$= 2\sqrt{7} + 6\sqrt{7}$

$= 8\sqrt{7}$.

b) $\sqrt{175} - \sqrt{63}$

$= \sqrt{7 \times 25} - \sqrt{7 \times 9}$

$= (\sqrt{7} \times 5) - (\sqrt{7} \times 3)$

$= 5\sqrt{7} - 3\sqrt{7}$

$= 2\sqrt{7}$