MATH 1115
Assignment 4

1(a) RTS $\lg 256 - \lg 64 + \lg 16 = 3\lg 4$

PROOF:

$$\lg 256 - \lg 64 + \lg 16 = \lg 4^4 - \lg 4^3 + \lg 4^2$$
$$= 4\lg 4 - 3\lg 4 + 2\lg 4$$
$$= \lg 4 + 2\lg 4$$
$$= 3\lg 4$$
□

1(b) $\lg_2 6 \times \lg_6 8$

change to base 10

$\lg_2 6 = \dfrac{\lg 6}{\lg 2}$ but $\lg_6 8 = \dfrac{\lg 8}{\lg 6}$

so, $$\lg_2 6 \times \lg_6 8 = \frac{\cancel{\lg 6}}{\lg 2} \times \frac{\lg 8}{\cancel{\lg 6}}$$
$$= \frac{\lg 8}{\lg 2}$$
$$= \frac{\lg 2^3}{\lg 2}$$
$$= \frac{3\cancel{\lg 2}}{\cancel{\lg 2}}$$
$$= 3$$
□

Alternative

change to base 2

$\lg_6 8 = \dfrac{\log_2 8}{\log_2 6}$ (i) Then $\log_2 6 \times \dfrac{\log_2 8}{\log_2 6} = \log_2 8$
$$= \lg_2 2^3$$
$$= 3\lg_2 2$$
$$= 3$$

2(a) $3^{2x} = 5^{x+1} \rightarrow \lg 3^{2x} = \lg 5^{x+1}$

$2x \lg 3 = (x+1)\lg 5$

$2x\lg 3 = x\lg 5 + \lg 5$

$2x\lg 3 - x\lg 5 = \lg 5$

$x(2\lg 3 - \lg 5) = \lg 5$

$x = \dfrac{\lg 5}{(2\lg 3 - \lg 5)}$

$x \approx 2.74$ (correct to 2 dp)

2(b) $\lg_2 5 + \lg_2 x = \lg_2 125$

$\lg_2 (5x) = \lg_2 125$

$5x = 125$

$x = 25$

2(c) $\ln(x+1) - \ln(2x) = \ln 4$

$\ln\left(\dfrac{x+1}{2x}\right) = \ln 4$

$\dfrac{x+1}{2x} = 4$

$x+1 = 8x$

$1 = 7x$

$\dfrac{1}{7} = x \Rightarrow x = 0.14$ (correct to 2dp)

2(d) $\lg(x+2)^3 = 9$

$3\lg(x+2) = 9$

$\lg(x+2) = 3$

$10^3 = x+2$ b/c $\lg = \lg_{10}$

$1000 = x+2$

$998 = x$

3(a) $\lg(x^3 y z^2) = \lg x^3 + \lg y + \lg z^2$

$= 3\lg x + \lg y + 2\lg z$

3(b) $\lg\left(\frac{x^3}{x^2 z}\right) = \lg\left(\frac{x}{z}\right) = \lg x - \lg z$

3(b) ALTERNATIVE

$\lg\left(\frac{x^3}{x^2 z}\right) = \lg x^3 - \lg(x^2 z)$

$= \lg x^3 - [\lg(x^2) + \lg z]$

[3] $= 3\lg x - 2\lg x - \lg z$

$= \lg x - \lg z$

3(c) $\lg\left(x\sqrt{\frac{y^2}{z^3}}\right) = \lg\left(\frac{xy}{z} \cdot \frac{1}{\sqrt{z}}\right) = \lg\left(\frac{xy}{z^{1.5}}\right)$

$\Rightarrow \lg xy - \lg z^{1.5} = \lg xy - 1.5\lg z$

$= \lg x + \lg y - 1.5\lg z$

3(c) ALTERNATIVE

$\lg\left(x\sqrt{\frac{y^2}{z^3}}\right) = \lg x + \lg\left(\frac{y^2}{z^3}\right)^{1/2}$

$= \lg x + 1/2 \lg\left(\frac{y^2}{z^3}\right)$

$= \lg x + 1/2 \{\lg y^2 - \lg z^3\}$

[5] $= \lg x + 1/2\{2\lg y - 3\lg z\}$

$= \lg x + \lg y - 3/2 \lg z$

4(a) $-x^2+2x=1$ (factorization by grouping)

$$-x^2+2x-1=0$$

$$-x^2+x+x-1=0$$

$$x(-x+1)-1(-x+1)=0$$

$$(x-1)(-x+1)$$

$$x=1$$

4(b) $5x^2-4x-2$ (by completing the square)

$a(x+h)^2+k$ where $h=\frac{b}{2a}$ and $k=c-\frac{b^2}{4a}$

so $h=\frac{-4}{2(5)} = \frac{-4}{10} = -\frac{2}{5}$

and $k=-2-\frac{(-4)^2}{(4)(5)} = -2-\frac{16}{20} = -\frac{14}{5}$

then

$$5\left(x-\frac{2}{5}\right)^2-\frac{14}{5}=0$$

$$5\left(x-\frac{2}{5}\right)^2=\frac{14}{5}$$

$$\cancel{5}\left(x-\frac{2}{5}\right)^2=\frac{14}{25}$$

$$\left(x-\frac{2}{5}\right)=\pm\sqrt{\frac{14}{25}}$$

$$x=2\pm\sqrt{\frac{14}{25}}$$

$x=1.15$ or $x=-0.35$

4(c) $5x^2+x-1=0$ (Qudratic formula)

$$x=\frac{-1\pm\sqrt{(1)^2-4(5)(-1)}}{2(5)}$$

$$=\frac{-1\pm\sqrt{1+20}}{10}$$

$$=\frac{-1\pm\sqrt{21}}{10}$$

$x=0.36$
(correct to 2dp)

$x=-0.56$
(correct to 2dp)

(5)(i) $pH = -\lg[H^+]$ where $H^+ = 2.4 \times 10^{-5} M$

$pH = -\lg[2.4 \times 10^{-5}]$

$= 4.62 M$ (correct 2dp)

(ii) $pH + pOH = 14$

But $pOH = 8.4$

$pH + 8.4 = 14$

$pH = 5.6$

Remember, $pH = -\lg[H^+]$

so, $5.6 = -\lg[H^+]$

$\lg([H^+]^{-1}) = 5.6$	$\lg([H^+]) = -5.6$
$10^{5.6} = [H^+]^{-1}$	$H^+ = 10^{-5.6}$
$10^{-5.6} = H^+$	

6(a)

$$
\begin{array}{r l}
 & x^2 + x + 2 \\
5x-2 \,) & 5x^3 + 3x^2 + 8x - 8 \\
 & -\;(5x^3 - 2x^2) \\
 & \quad 5x^2 + 8x \\
 & \quad -\;(5x^2 - 2x) \\
 & \qquad 10x - 8 \\
 & \qquad -\;(10x - 4) \\
 & \qquad\qquad -4
\end{array}
$$

$$(x^2 + x + 2) - \frac{4}{5x-2}$$

6(b)

$$
\begin{array}{r l}
 & x^2 + 0 - 16 \\
x-3 \,) & x^3 - 3x^2 - 16x + 48 \\
 & -\;(x^3 - 3x^2) \\
 & \quad 0x^2 - 16x \\
 & \quad 0x^2 + 0 \\
 & \qquad -16x + 48 \\
 & \qquad -\;(16x + 48) \\
 & \qquad\qquad 0
\end{array}
$$

hence $(x-3)$ is a factor of $x^3 - 3x^2 - 16x + 48$

so $x^3 - 3x^2 - 16x + 48 = (x-3)(x^2-16) = 0$

$(x-3)(x^2-16) = 0 \Rightarrow (x-3)(x-4)(x+4) = 0$

$x = 3 \quad x = \pm 4$ [3]

6

7(a)

$$\begin{array}{r} 4x^2+3x+4 \\ x-1 \,\big)\, \overline{4x^3-x^2+x-2} \\ -\;4x^3-4x^2 \\ \hline 3x^2+x \\ -\;3x^2-3x \\ \hline 4x-2 \\ -\;4x-4 \\ \hline 2 \end{array}$$

[5]

$f(1) = 4(1)^3-(1)^2+(1)-2$

$= 4-1+1-2$

$= 2$

7(b)

$$\begin{array}{r} 2x^2-\frac{3}{2}x+\frac{5}{4} \\ 2x+1 \,\big)\, \overline{4x^3-x^2+x-2} \\ -\;4x^3+2x^2 \\ \hline -3x^2+x \\ -\;-3x^2-\frac{3}{2}x \\ \hline \frac{5}{2}x-2 \\ -\;\frac{5}{2}x+\frac{5}{4} \\ \hline \frac{13}{4} \end{array}$$

$f(-1/2) = 4(-1/2)^3-(-1/2)^2+(-1/2)-2$

$= 4(-1/8)-\frac{1}{4}-\frac{1}{2}-2$

$= \frac{-13}{4}$