Assignment 5 $\underline{\underline{75}}$

(1) Recall: y radians $= \frac{180^\circ}{\pi} \times y$

(a) $\frac{3}{2}\pi^c$

$= \frac{180^\circ}{\pi} \times \frac{3}{2}\pi$ [1]

$= 180^\circ \times \frac{3}{2}$ $\underline{\underline{[2]}}$

$= 270^\circ$ [1]

(b) $\frac{8}{9}\pi^c$

$= \frac{180^\circ}{\pi} \times \frac{8}{9}\pi$ [1]

$= 180^\circ \times \frac{8}{9}$ $\underline{\underline{[2]}}$

$= 160^\circ$ [1]

(c) $\frac{7}{6}\pi^c$

$= \frac{180^\circ}{\pi} \times \frac{7}{6}\pi$ [1]

$= 180 \times \frac{7}{6}$ $\underline{\underline{[2]}}$

$= 210^\circ$ [1]

(2) Recall: $x^\circ = \frac{\pi}{180} \times x$ radians

(a) 120°

$= \frac{\pi}{180} \times \frac{120}{1} = \frac{2}{3}\pi^c$ [1] [1] [2]

(b) 185°

$= \frac{\pi}{180} \times \frac{185}{1} = \frac{37}{36}\pi^c$ or $1.028\pi^c$ (correct to 3dp) [1] [1] [2]

(3) Factor Thm

$x+5=0$
$x=-5$

then
$f(-5) = (-5)^3 + 9(-5)^2 + 23(-5) + 15$
$= -125 + 225 - 115 + 15$
$= 0$ [2]

Hence $(x+5)$ is a factor of $x^3+9x^2+23x+15$

$$
\begin{array}{r|l}
 & x^2 + 4x + 3 \quad [1] \\
\hline
x+5 & x^3 + 9x^2 + 23x + 15 \\
 & -(\cancel{x^3} + 5x^2) \quad [1] \\
\hline
 & 4x^2 + 23x \\
 & -(\cancel{4x^2} + 20x) \quad [1] \\
\hline
 & 3x + 15 \\
 & -(\cancel{3x} + \cancel{15}) \quad [1] \\
\hline
 & 0 \quad [1]
\end{array}
$$

then

$x^3 + 9x^2 + 23x + 15 = (x+5)(x^2+4x+3)$

$x^3 + 9x^2 + 23x + 15 = (x+5)(x^2 + 4x + 3)$ [1]

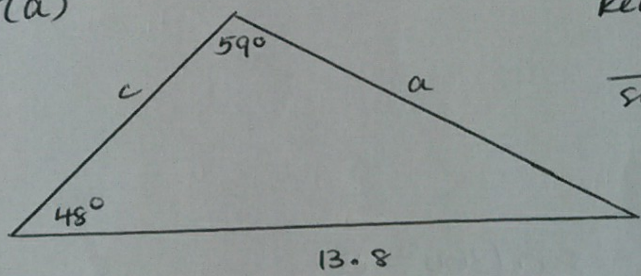
$= (x+5)(x^2 + x + 3x + 3)$ [1]

$= (x+5)\,[x(x+1) + 3(x+1)]$ [1]

$= (x+5)(x+3)(x+1)$ [1]

[11]

4(a)



Recall: sine Rule

$$\frac{a}{\sin \hat{A}} = \frac{b}{\sin \hat{B}} = \frac{c}{\sin \hat{C}}$$

using sine Rule

$$\frac{13.8}{\sin(59^\circ)} = \frac{a}{\sin(48^\circ)}$$ [1]

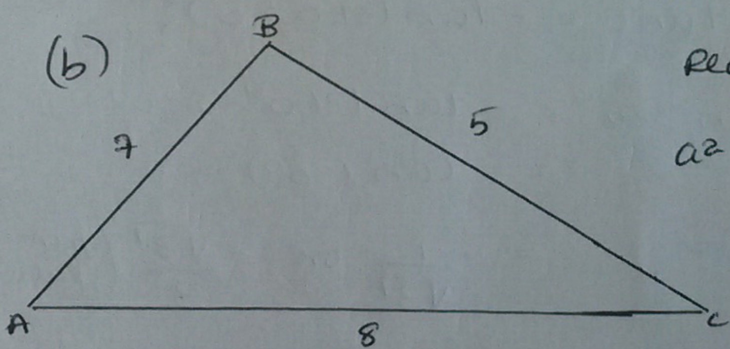
$\Rightarrow 13.8 \times \sin(48^\circ) = a \times \sin(59^\circ)$ [1]

$\Rightarrow \dfrac{13.8 \times \sin(48^\circ)}{\sin(59^\circ)} = a$ [1]

[4]

$\Rightarrow a = 11.96$ (correct to 2dp) [1]

(b)



Recall: cosine Rule

$a^2 = b^2 + c^2 - (2bc \cos \hat{A})$

using cosine Rule

$5^2 = 8^2 + 7^2 - (2 \times 8 \times 7 \times \cos(\hat{A}))$ [1]

$25 = 64 + 79 - 112 \cos \hat{A}$ [1]

$25 = 113 - 112 \cos \hat{A}$ [1]

$25 - 113 = -112 \cos \hat{A}$ [1]

$-88 = -112 \cos \hat{A}$ [1]

$\dfrac{88}{112} = \cos \hat{A}$ [1]

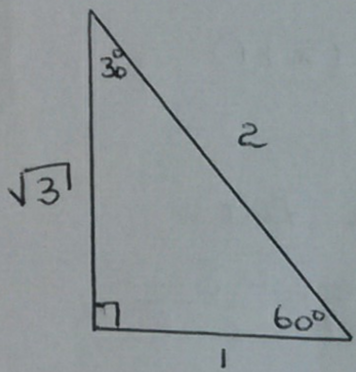
$\Rightarrow \hat{A} = \cos^{-1}\left(\dfrac{88}{112}\right) = 38.2^\circ$ [1]

[8]

(5) (a) Recall : $\sin(180^\circ - \theta) = \sin\theta$

then

$$\sin 120^\circ = \sin(180^\circ - 120^\circ) \quad [1]$$



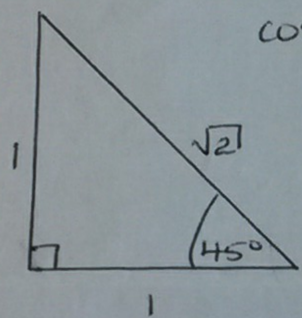
$$= \sin(60^\circ) \quad [1]$$

$$= \frac{\sqrt{3}}{2} \quad [1]$$

[3]

(b) Recall: $\cos(\theta) = \sin(360^\circ - \theta)$

then



$$\cos(315^\circ) = \sin(360^\circ - 315^\circ) \quad [1]$$

[3]

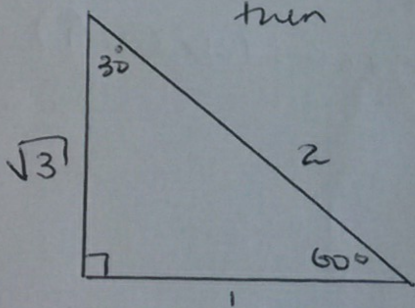
$$= \cos(45^\circ) \quad [1]$$

$$= \frac{1}{\sqrt{2}} \text{ or } \frac{\sqrt{2}}{2}$$ (from Rationalizing) [1]

(c) Recall : $\tan(360^\circ - \theta) = -\tan\theta$

$$\Rightarrow \tan\theta = -\tan(360^\circ - \theta)$$

then



$$\tan 330^\circ = -\tan(360^\circ - 330^\circ) \quad [1]$$

$$= -\tan(30^\circ) \quad [1]$$

[3]

$$= -\frac{1}{\sqrt{3}} \text{ or } -\frac{\sqrt{3}}{3}$$ (from Rationalizing)

[1]

(d) $\sin 45^\circ \times \operatorname{cosec} 45^\circ$

Recall $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$ [1]

[2]

$$\Rightarrow \cancel{\sin 45^\circ} \times \frac{1}{\cancel{\sin 45^\circ}} = 1 \quad [1]$$

6(a) $\sin\theta = -0.6325$

$\theta = \sin^{-1}(0.6325)$ [1]

$\theta_1 = 39.2^\circ$ [1]

But sin is -ve in the 3rd + 4th Quadrants

3rd Quadrant

$\theta_3 = 180^\circ + 39.2^\circ = 219.2^\circ$ [1]

4th Quadrant [4]

$\theta_4 = 360^\circ - 39.2^\circ = 320.8^\circ$ [1]

therefore, solutions: $\theta = 219.2^\circ, 320.8^\circ$

(b) $\tan\theta - 3.4 = 0$

$\tan\theta = 3.4$ [1]

$\theta = \tan^{-1}(3.4)$ [1]

$\theta_1 = 73.6^\circ$ [1]

But tan is +ve in the 1st + 3rd Quadrants

1st Quadrant

$\theta_1 = 73.6^\circ$

3rd Quadrant [4]

$\theta_3 = 180^\circ + 73.6^\circ$ [1]

$\theta_3 = 253.6^\circ$

therefore, solutions: $\theta = 73.6^\circ, 253.6^\circ$

6(c) $2\cos^2\theta = 1 - 2\sin\theta$

But $\cos^2\theta + \sin^2\theta = 1$

then $\cos^2\theta = 1 - \sin^2\theta$

$2(1-\sin^2\theta) = 1 - 2\sin\theta$ [1]

$2 - 2\sin^2\theta = 1 - 2\sin\theta$ [1]

$2 - 2\sin^2\theta - 1 + 2\sin\theta = 0$ [1]

$-2\sin^2\theta + 2\sin\theta + 1 = 0$ [1]

let $\sin\theta = x$

$-2x^2 + 2x + 1 = 0$ (solve using the Quadratic Formula)

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(-2)(1)}}{2(-2)}$$ [1]

$$x = \frac{-2 \pm \sqrt{4+8}}{-4}$$ [1]

$$x = \frac{-2 \pm \sqrt{12}}{-4}$$ [1]

$x = -0.366$ $x = 1.366$

then

$\sin\theta = -0.366$

$\theta = \sin^{-1}(0.366)$ [1]

$\theta = 21.5$ [1]

$\sin\theta = 1.366$

$\downarrow$ [1]

undefined b/c

$-1 \le \sin\theta \le 1$

But sin is -ve in the 3rd + 4th Quadrants

3rd Quadrant

$\theta_3 = 180^\circ + 21.5^\circ = 201.5^\circ$ [1]

[12]

4th Quadrant

$\theta_4 = 360^\circ - 21.5^\circ = 338.5^\circ$ [1]

Therefore, solutions: $\theta = 201.5^\circ, 338.5^\circ$

7(a) $2\sin(x) = 1$

$\sin x = 1/2$ [1]

$x = \sin^{-1}(1/2)$ [1]

$x = 0.524^c$ [1]

But sine is +ve in the 1st + 4th Quadrants

1st Quadrant

$x_1 = 0.524^c$

2nd Quadrant

[4]

$x_2 = \pi^c - 0.524^c$ [1]

$x_2 = 2.61^c$

therefore, solutions: $x = 0.524^c, 2.61^c$

(b) $\sin x + \sqrt{2} = -\sin x$

$\sin x + \sin x + \sqrt{2} = 0$ [1]

$2\sin x = -\sqrt{2}$ [1]

$\sin x = \dfrac{-\sqrt{2}}{2}$ [1]

$x = \sin^{-1}\left(\dfrac{\sqrt{2}}{2}\right)$

[1]

$x = 0.785$ [1]

But sin is -ve in the 3rd + 4th Quad's

3rd Quadrant

$x_3 = \pi + 0.785$

$x_3 = 3.927^c$ [1]

4th Quadrant

$x_4 = 2\pi - 0.785$

$x_4 = 5.498$ [1]

therefore solutions: $x = 3.927^c, 5.498^c$

[7]