INFERENTIAL STATISTICS

Hypothesis Testing

Population vs Sample

Psycholinguistics is interested in how people process certain linguistic phenomena, but our experiments cannot collect data on every person and every instance, so instead we select a subset of people and instances to study

Population

- A complete set of individuals, events, utterances, etc.
- All German speakers
- All German sentences containing S (+s, -s) and K (+k, -k)

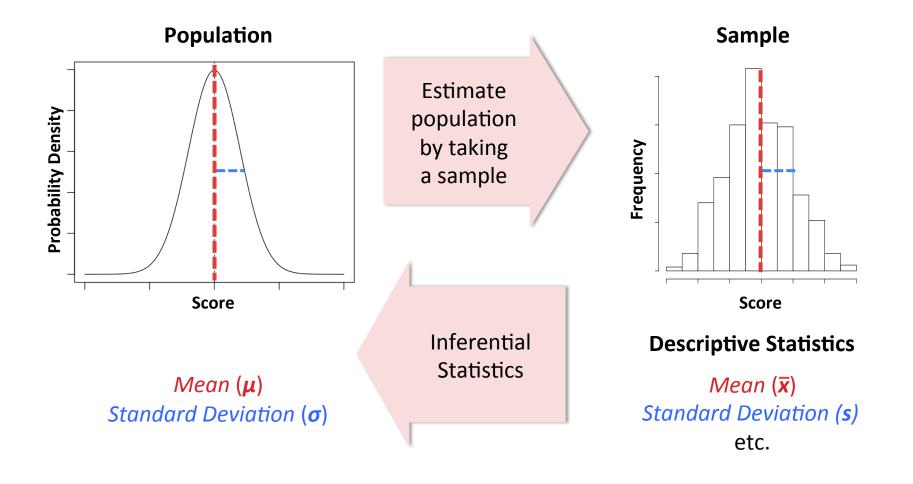
Estimate population by taking a sample

Sample

- A subset of a population
- Usually presumed to be randomly selected
- Central problem is uncertainty about how similar the sample is to the true population

Population vs Sample

Psycholinguistics is interested in how people process certain linguistic phenomena, but our experiments cannot collect data on every person and every instance, so instead we select a subset of people and instances to study



Sampling in Psycholinguistics

In psycholinguistic experiments we sample from:

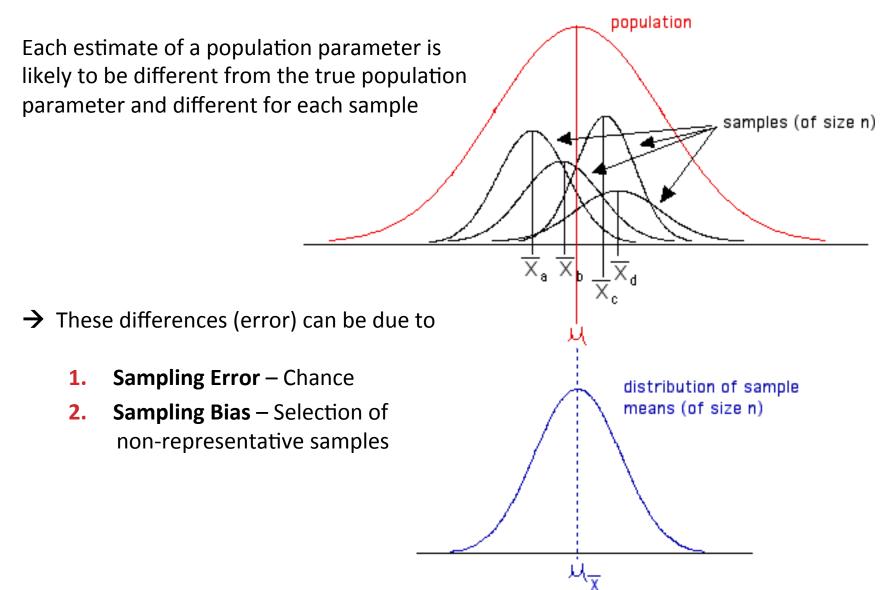
1) Speakers

Analysis *by Subjects*: We use inferential statistics to generalize the results to the population of all speakers of a language (not just the participants used in experiment)

2) Language

Analysis *by Items*: We use inferential statistics to generalize the results to the entire collection of linguistic items displaying the phenomenon of interest (not just the items used in experiment)

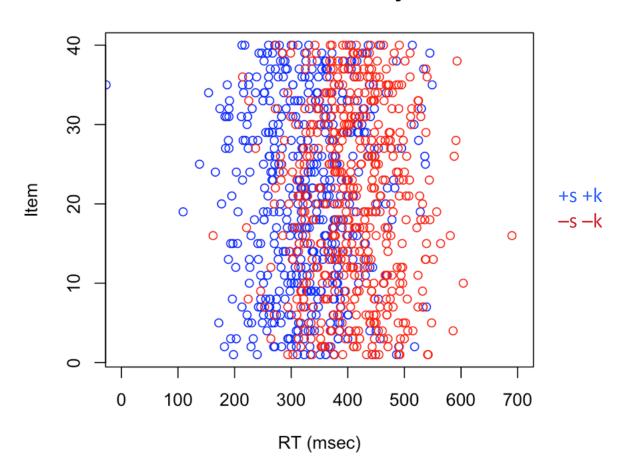
Sampling Error and Bias

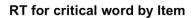


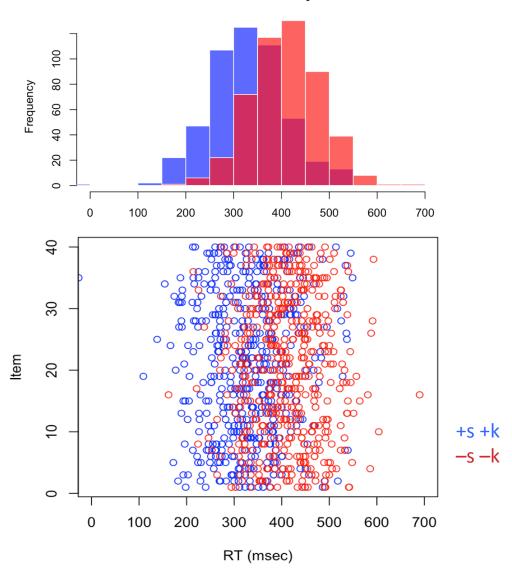
Self-paced reading version of our study

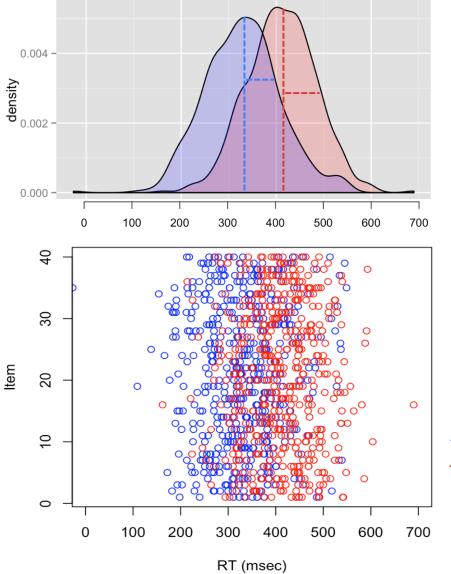
- 2 x 2 within-subjects design
 - Stereotype Consistency (+stereotypical, –stereotypical)
 - Speaker-specific Knowledge Consistency (+knowledge, -knowledge)
- DV: RT for critical word
- Let's say we will run 50 participants
- And each participant will see 40 items
- → How many data points will we have after running this experiment?
 - Per subject?
 - Per condition?

RT for critical word by Item









Are the differences between conditions significantly greater than what we would expect to see between any two samples drawn from the same population?

→ Need statistical analysis

How do we analyze our dataset?

- 1. Organize it
- 2. Summarize it
- 3. Apply statistical test

Analysis by Subjects

Subj	+s +k	-s -k
1	312ms	325ms
2	365ms	356ms
3	200ms	224ms
4	324ms	388ms
5	356ms	412ms
6	326ms	378ms
7	27 9ms	299ms
•••		
50	323ms	340ms
X	320	350
S	48	55

1. Organize: Aggregate data by subjects

2. Summarize: Descriptive statistics

Statistical Hypotheses Testing

To test whether a difference is significant, we first make the assumption that it is *not* (i.e., that the difference is just due to chance)

Null hypothesis (H₀)

$$\mu_{+s+k} = \mu_{-s-k} \implies \mu_{+s+k} - \mu_{-s-k} = 0$$

The research hypothesis, the outcome we predict, is that there is a *true* difference (i.e., that the difference is due to the manipulation)

Alternative hypothesis (H₁)

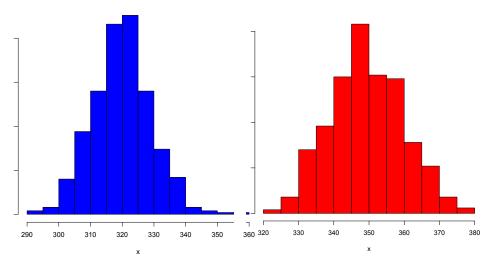
a)
$$\mu_{+s+k} \neq \mu_{-s-k} \Rightarrow \mu_{+s+k} - \mu_{-s-k} \neq 0$$
 Two-tailed hypothesis

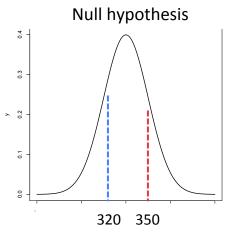
b)
$$\mu_{+s+k} - \mu_{-s-k} < 0$$
 One-tailed hypothesis

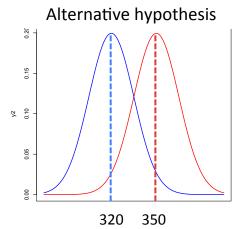
 \rightarrow Goal: See if we can reject H₀, thereby supporting (BUT NOT PROVING) H₁

3. Apply statistical test

Subj	+s +k	-s -k
1	312ms	325ms
2	365ms	356ms
3	200ms	224ms
4	324ms	388ms
5	356ms	412ms
6	326ms	378ms
7	279ms	299ms
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X	320	350
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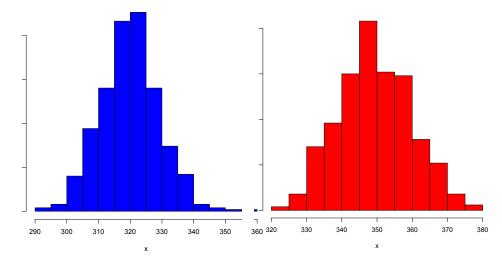


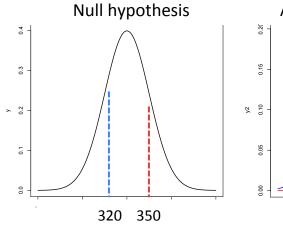
3. Apply statistical test

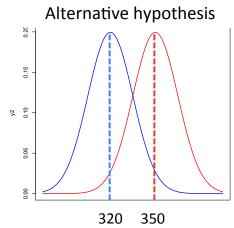
Statistical tests (e.g., *t*-test, ANOVA) tell us how likely it is to observe the difference, **assuming that the null hypothesis is true**

If this probability (p-value) is low (e.g., < 5%) then we can reject H₀

- → The difference is significant
 - The same difference would likely be observed if we used different subjects (or items)
 - Thus, the result generalizes to the population







Statistical Tests

What kind of statistical test should be used?

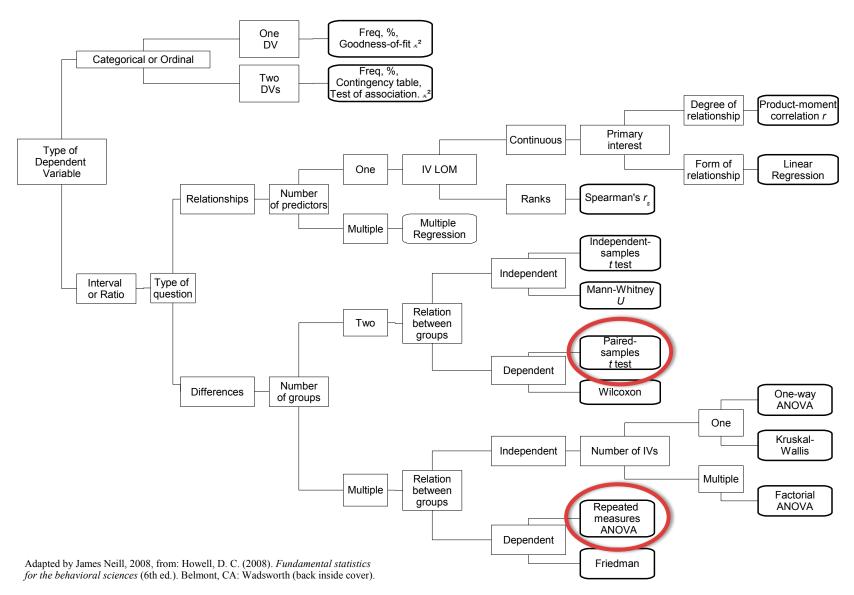
• t-test, ANOVA, χ^2 -test, mixed effects models, etc.

Depends on

- Type of DV data (Continuous vs Categorical)
- Number of IVs in the design
- The assumed underlying distributions (normal, binomial, etc.)
- Whether design is between-subjects or within-subjects

In psycholinguistics, statistical analyses are performed both by-subjects (t_1, F_1) and by-items (t_2, F_2)

Decision Tree



Test types and what they tell us

Overview of two most common types of test in psycholinguistics

t-tests

- Can only compare the effect of 1 IV with 2 levels
- Within-subjects: "Paired-samples" t-test (matched, related)
- Between-subjects: "Independent samples" *t*-test

Analysis of Variance (ANOVA)

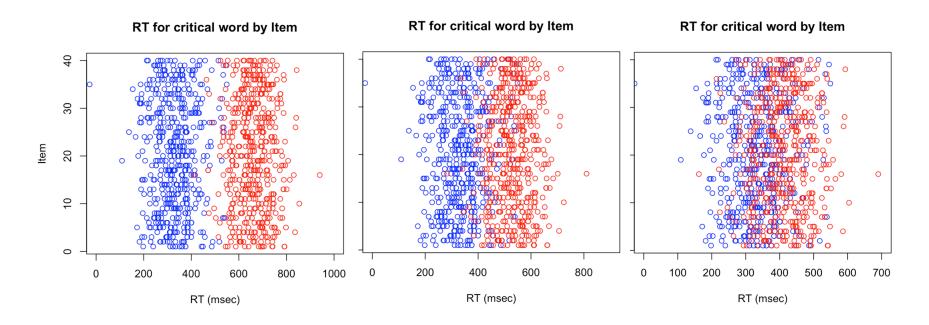
- Can do everything a t-test can do and more
- Can compare more than 2 IVs (factors) and more than 2 levels
- Can test for independent effects of each factor, even when factors have multiple levels
- Can test for interactions (i.e., relationships between factors)

Mixed effects models (LMER, GLMER)

Next time

Variability Between and Within Conditions

Most statistical tests calculate a ratio that takes into account both sources of variability



Rule of thumb

If variability between conditions is large and variability within conditions (or groups) is relatively small, then the difference between conditions is likely to be significant

The *t*-test

Tests whether a difference between two means is significant

Simplified formula for paired-samples *t*-test (for repeated measures design)

$$t = \frac{(\overline{x_1} - \overline{x_2}) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

Variability between conditions (signal)

Variability within conditions (noise)

Rule of thumb

If variability within conditions (or groups) is relatively small and variability between conditions is large, then the difference between conditions is likely to be significant

3. Apply statistical test

Subj	+s +k	-s -k
1	312ms	325ms
2	365ms	356ms
3	200ms	224ms
4	324ms	388ms
5	356ms	412ms
6	326ms	378ms
7	279ms	299ms
•••	•••	•••
50	323ms	340ms
\overline{x}	320	350
S	48	55

$$t = \frac{(\overline{x_1} - \overline{x_2}) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$$=\frac{-30}{\sqrt{(2304/50)+(3025/50)}}=$$

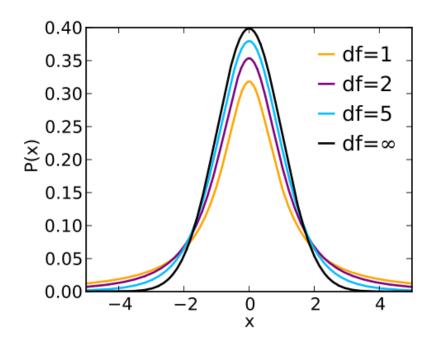
$$=\frac{-30}{\sqrt{46.08+106.58}}=-2.91$$

If the probability of observing **t = -2.91** is low, then we can reject the null hypothesis

How do we know if probability of t is low?

Look at the t**-distribution** (i.e., the distribution of all possible t-values if H_0 is true)

- t-values are normally distributed with $\mu = 0$
- Large or small values of t are rare, but values close to 0 are common



Exact shape of *t*-distribution depends on the number of **degrees of freedom** (*df*)

As **df** goes to infinity, the t-distribution converges to standard normal distribution ($\mu = 0$, $\sigma = 1$)

$$df = N - 1$$

$$df = 50 - 1 = 49$$

Degrees of Freedom (df)

Defined as number of values in calculation of statistic that are "free to vary"

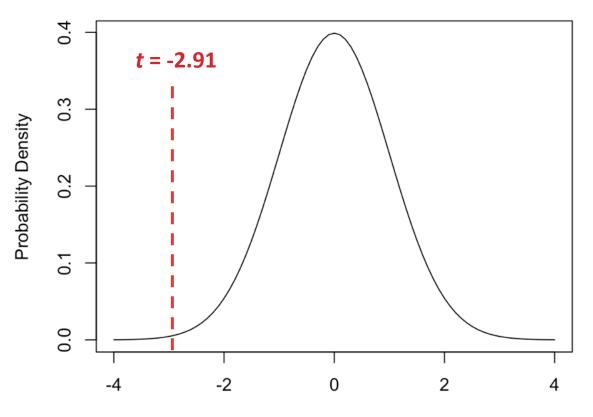
- Imagine you have four numbers (a, b, c, d) that must add up to X
- You are free to choose the first three numbers at random, but the fourth must be chosen so that it makes the total equal to X
- Thus, df = 3

→ df provides a more accurate estimate of population parameters

- Uncertainty about the true standard deviation requires us to qualify our beliefs (i.e., smaller sample sizes require us to be more conservative)
- As our sample size increases, df increases, which lowers the threshold for significance

t-distribution

t-distribution with 49 df

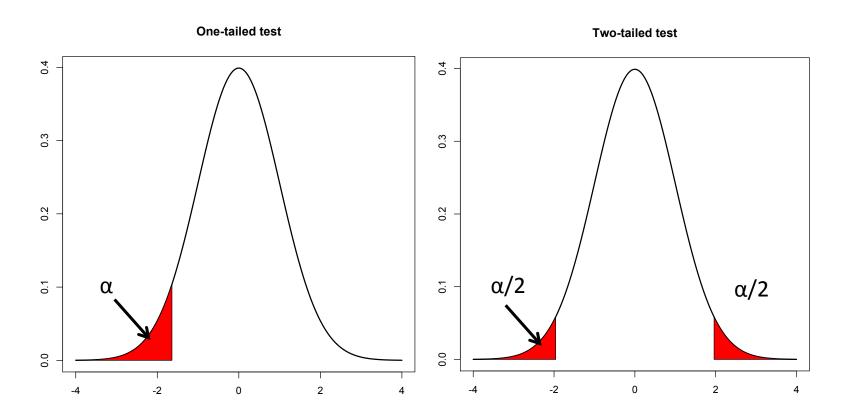


- \rightarrow Can we reject H₀?
 - Depends on the "critical value" of t (t_{crit})
 - Which depends on our significance threshold (α)

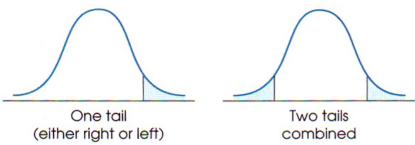
Significance Level: α

Alpha (α) – arbitrary cutoff value representing probability with which we are willing to reject H₀ when it is, in fact, true

 α levels conventionally used: .05, .01 $\,$



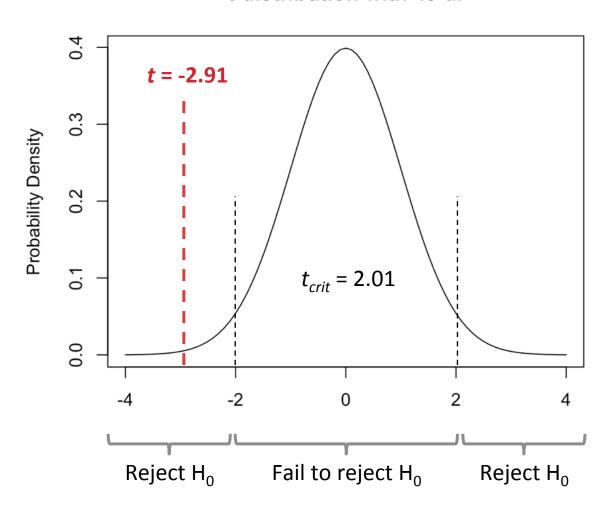
The *t*-table



			PROPORTION IN	ONE TAIL		
	0.25	0.10	0.05	0.025	0.01	0.005
		PRO	OPORTION IN TWO T	AILS COMPINED		
df	0.50	0.20	0.10	0.05	0.02	0.01
1	1.000	3.078	6.314	12.706	31.821	63.657
2	0.816	1.886	2.920	4.303	6.965	9.925
3	0.765	1.638	2.353	3.182	4.541	5.841
4	0.741	1.533	2.132	2.776	3.747	4.604
5	0.727	1.476	2.015	2.571	3.365	4.032
6	0.718	1.440	1.943	2.447	3.143	3.707
7	0.711	1.415	1.895	2.365	2.998	3.499
8	0.706	1.397	1.860	2.306	2.896	3.355
9	0.703	1.383	1.833	2.262	2.821	3.250
10	0.700	1.372	1.812	2.228	2.764	3.169
11	0.697	1.363	1.796	2.201	2.718	3.106
12	0.695	1.356	1.782	2.179	2.681	3.055
13	0.694	1.350	1.771	2.160	2.650	3.012
14	0.692	1.345	1.761	2.145	2.624	2.977
15	0.691	1.341	1.753	2.131	2.602	2.947
16	0.690	1.337	1.746	2.120	2.583	2.921
17	0.689	1.333	1.740	2.110	2.567	2.898
18	0.688	1.330	1.734	2.101	2.552	2.878
40	0.681	1.303	1.684	2.021	2.423	2.704
60	0.679	1.296	1.671	2.000	2.390	2.660
120	0.677	1.289	1.038	1.980	2.358	2.617
oc ·	0.674	1.282	1.645	1.960	2.326	2.576

t-distribution

t-distribution with 49 df



Paired-samples t-test

Calculating t-statistic and associated p-value in R (2-tail test)

```
# aggregated data (by subjects or by items) for contrast of interest
cw.data.2conds <- subset(HypData, Word.number=="cw" & (Cond=="ps.pk" | Cond=="ms.mk"))</pre>
                      # DV ~ IV (in this example, only 2 levels of Cond)
t.test(Word.RT ~ Cond.
         data = cw.data.2conds,  # name of the dataset
         paired = TRUE,  # are the groups paired or not?
                        # difference between means under H<sub>e</sub>
         mu = 0
         Paired t-test
data: Word.RT by Cond
t = -2.906, df = 49, p-value = 0.003382
alternative hypothesis: true difference in means is not equal to 0 # default = 2-tail
95 percent confidence interval: # true difference between means is likely to fall
 -57.59908 -12.12092 # between -57.6 and -12.12
sample estimates:
mean of the differences
                -34.86
```

Paired-samples t-test

Calculating t-statistic and associated p-value in R (1-tail test)

```
# aggregated data (by subjects or by items) for contrast of interest
cw.data.2conds <- subset(HypData, Word.number=="cw" & (Cond=="ps.pk" | Cond=="ms.mk"))</pre>
t.test(Word.RT ~ Cond.
                       # DV ~ IV (in this example, only 2 levels of Cond)
         data = cw.data.2conds,  # name of the dataset
         paired = TRUE,  # are the groups paired or not?
         alternative = "less", # direction of H<sub>1</sub>
         mu = 0)
                                  # true difference between means under H<sub>e</sub>
         Paired t-test
data: Word.RT by Cond
t = -2.906, df = 49, p-value = 0.001691
alternative hypothesis: true difference in means is less than 0 # 1-tail
95 percent confidence interval: # true difference between means is likely to fall
     -Inf -15.88921 # between -Infinity and -15.9
sample estimates:
mean of the differences
                -34.86
```

Effect Size

A statistical estimate of the magnitude of the effect that is relatively independent of sample size, and thus allows us to compare across studies

- Calculated after rejecting the null hypothesis (otherwise, it has no meaning)
- The larger the effect size, the fewer subjects needed to detect the effect
- Simple method: **r**² (coefficient of determination, "r-squared")

$$r^2 = \frac{t^2}{t^2 + df}$$

$r^2 = \frac{-2.91^2}{-2.91^2 + 49}$

$r^2 = 0.1474$

By convention:

0.01 small effect

0.09 medium effect

0.25 large effect

→ 14.8% of the variance in RT is predictable from the IV

3. Apply statistical test

Subj	+s +k	-s -k
1	312ms	325ms
2	365ms	356ms
3	200ms	224ms
4	324ms	388ms
5	356ms	412ms
6	326ms	378ms
7	27 9ms	2 99ms
•••	•••	•••
50	323ms	340ms
X	320	350
S	48	55

- 1. Choose the alpha level (e.g., .05)
- 2. Calculate t score
- 3. Compare to t_{crit}

If
$$|t| > t_{\alpha}$$
 then $p < .05$

- The difference is significant
- Reject H₀

If
$$|t| \le t_{\alpha}$$
 then $p \ge .05$

- Null result
- Fail to reject H₀
- 4. If H₀ rejected, calculate effect size

Reporting the Results

Report all of the following:

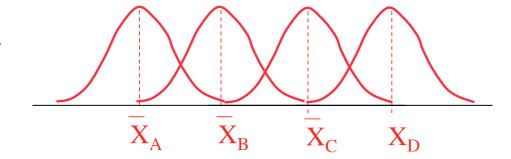
- The observed difference between conditions
- The specific kind of test (e.g., t-test)
- The computed statistic (e.g., t)
- Degrees of freedom for the test
- The p-value of the test
- The effect size (e.g., r²)

"The mean response time for critical words in the +s+k condition was 30 ms faster than for the –s-k condition. A repeated measures t-test yielded a significant difference, t(49) = -2.91, p < .01, $r^2 = 0.147$."

Analysis of Variance (ANOVA)

Tests the effect of a categorical IV on a continuous DV

Test whether means of two or more conditions are significantly different from each other



Rule of thumb

If variability between conditions is large and variability within conditions (or groups) is relatively small, then the difference between conditions is likely to be significant

Analysis of Variance (ANOVA)

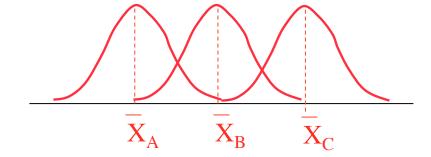
Categorized by number of IVs and whether groups are independent or dependent:

Type of ANOVA	Characteristics	With 2 levels, equivalent to:
One-way independent	One IV with two or more levelsEach observation is independent	→ Independent samples <i>t</i> -test
One-way repeated measures	 One IV with two or more levels Multiple observations from the same subjects 	→ Paired-samples t-test
Two-way independent	Two IVs (factors) with two or more levelsEach observation is independent	
Two-way repeated measures	 Two factors with two or more levels Multiple observations from the same subjects 	Factorial ANOVA
		AIVOVA

- The term "way" describes number of IVs
- Factorial ANOVAs test how individual IVs and/or interactions affect the DV

Hypothesis Testing

e.g., One way ANOVA with 3 levels



Null hypothesis:

 H_0 : all the groups are equal

$$\overline{X_A} = \overline{X_B} = \overline{X_C}$$

Alternative hypothesis:

H₁: not all the groups are equal

$$\overline{X_A} \neq \overline{X_B} \neq \overline{X_C}$$
 $\overline{X_A} \neq \overline{X_B} = \overline{X_C}$

$$\overline{X_A} \neq \overline{X_B} = \overline{X_C}$$

$$\overline{X_{\Delta}} = \overline{X_{B}} \neq \overline{X_{C}}$$
 $\overline{X_{\Delta}} = \overline{X_{C}} \neq \overline{X_{B}}$

$$\overline{X_A} = \overline{X_C} \neq \overline{X_B}$$

- \rightarrow ANOVA tests H₀ Is there a difference between any one of these groups from another that is unlikely to be due to chance?
 - If probability of F is low, we can reject H₀
 - Then do follow-up pairwise comparisons

Test 1:
$$A \neq B$$

Test
$$3: B = 0$$

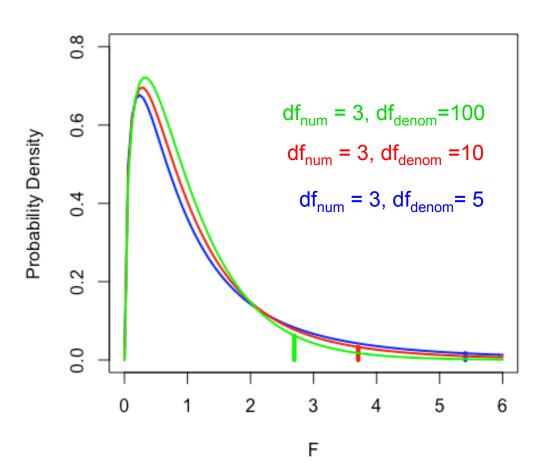
p-values corrected for hen do follow-up pairwise comparisons

Test 1: $A \neq B$ Test 2: $A \neq C$ Test 3: B = Cmultiple comparisons (e.g., Bonferroni correction)

How do we know if probability of *F* is low?

Look at the F**-distribution** (i.e., the distribution of all possible F-values if H_0 is true)

• Shape of F distribution depends on two degrees of freedom (df_{num}, df_{denom})



$$F = \frac{\text{Variability between conditions (signal)}}{\text{Variability within conditions (noise)}}$$

$$\frac{df_{num} = k - 1}{k = number\ of\ conditions\ /\ groups}$$

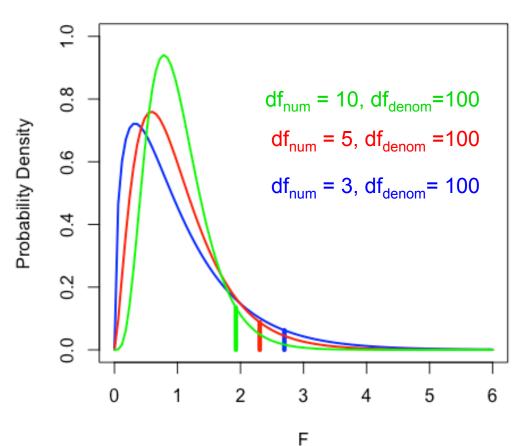
$$\frac{df_{denom}}{denom} = (n_{total} - 1)(k - 1)$$

$$n_{total} = total\ number\ of\ scores$$

How do we know if probability of F is low?

Look at the F**-distribution** (i.e., the distribution of all possible F-values if H_0 is true)

• Shape of F distribution depends on two degrees of freedom (df_{num} , df_{denom})



$$F = \frac{\text{Variability between conditions (signal)}}{\text{Variability within conditions (noise)}}$$

$$\frac{df_{num} = k - 1}{k = number\ of\ conditions\ /\ groups}$$

$$\frac{df_{denom}}{denom} = (n_{total} - 1)(k - 1)$$

 $n_{total} = total \ number \ of \ scores$

3. Apply statistical test

Subj	+s +k	
1	312ms	325ms
2	365ms	356ms
3	200ms	224ms
4	324ms	388ms
5	356ms	412ms
6	326ms	378ms
7	279ms	299ms
•••		•••
50	323ms	340ms
X	320	350
S	48	55

- 1. Choose the alpha level (e.g., .05)
- 2. Calculate *F*-statistic, *p*-value, and effect size
- 3. If more than 2 levels within an IV, conduct follow-up pairwise comparisons with p-values corrected for multiple comparisons (e.g., Bonferroni correction)

Hypothetical Data

3. Apply statistical test

Subj	+s +k	-s -k	
1	312ms	325ms	
2	365ms	356ms	
3	200ms	22 4ms	
4	324ms	388ms	
5	356ms	412ms	
6	326ms	378ms	
7	279ms	299ms	
•••			
50	323ms	340ms	
X	320	350	
5	48	55	

$$F = \frac{\sum n_{in\ a\ group} * (\overline{x}_{group} - \overline{x}_G)^2}{\sum (x_i - \overline{x}_{group} - (\overline{x}_{subject} - \overline{x}_{group}))^2}$$

$$n_{total} - k - (n_{subjects} - 1)$$

k = number of conditions / groups $n_{total} = total \ number of scores$ $\overline{x}_G = grand \ mean$ $n_{in \ a \ group} = number of scores in each group$ $\overline{x}_{group} = mean \ of \ the \ group \ the \ score \ comes \ from$ $x_i = individual \ score$

Calculating F-statistic, p-value, and effect size (GES) in R using ezANOVA()

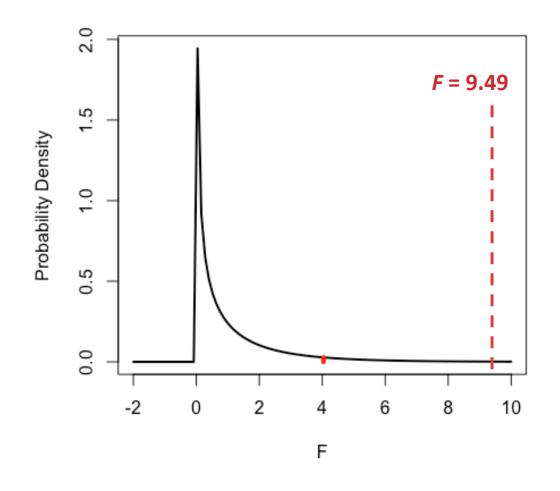
\$ANOVA Effect DFn DFd F p p<.05 ges Cond 1 49 9.491118 0.003382163 * 0.1055792

Generalized eta-squared (GES)

0.02 small effect0.13 medium effect0.26 large effect

→ The larger the effect size, the fewer subjects needed to detect the effect





→ We can reject the null hypothesis

Paired-samples t-test

Calculating t-statistic and associated p-value in R

t.test(Word.RT ~ Cond, # DV ~ IV

-34.86

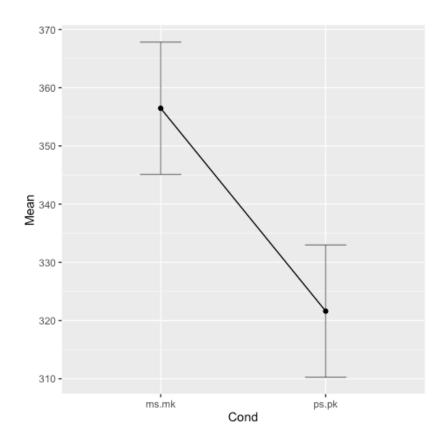
```
data = cw.data.2conds,  # name of the dataset
        # difference between means under H<sub>o</sub>
        mu = 0)
        Paired t-test
                                    t^2 = F = 9.49
data: Word.RT by Cond
t = -2.906, df = 49, p-value = 0.003382
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -57.59908 -12.12092
sample estimates:
mean of the differences
```

Calculate descriptive statistics using ezStats()

get table of descriptive statistics

2 ps.pk 50 321.62 44.40762 22.73908

Plot results using ezPlot()



Factorial ANOVA

Test whether individual factors and/or their interactions affect the DV

- Two or more factors (IVs)
 - Factors may be within or between
 - Overall design may be entirely within, entirely between, or mixed
- Multiple F-ratios are computed
 - One to test the main effect each factor:

```
The effect of one IV on the DV, ignoring the effects of all other IVs

Stereotypicality (+s, -s)

Knowledge (+k, -k)
```

One to test each potential interaction:

Whether the impact of one IV differs depending on value of another IV Stereotypicality x Knowledge (Stereotypicality:Knowledge)

3. Apply statistical test

Subj	+s +k	+s –k	-s +k	-s -k
1	312	333	341	325
2	365	389	368	356
3	200	221	277	224
4	324	312	365	388
5	356	367	399	412
6	326	399	387	378
7	279	295	296	299
•••		•••	•••	
50	323	333	339	340
Х	320	338	347	350
s	48	63	61	55

- 1. Choose the alpha level (e.g., .05)
- 2. Calculate *F*-statistic, *p*-value, and effect size
- 3. If more than 2 levels within an IV, conduct follow-up pairwise comparisons with *p*-values corrected for multiple comparisons (e.g., Bonferroni correction)

Calculating F-statistic, p-value, and effect size in R using ezANOVA()

```
# aggregate data (by subjects or by items) for contrasts of interest
cw.data.allConds <- subset(HypData, Word.number=="cw")</pre>
m2 <- ezANOVA(data = cw.data.allConds,  # name of dataframe</pre>
               dv = Word.RT
                                             # column name of DV
                                             # column name of within-group index
               wid = Subj,
               within = .(Stereotypicality, # cols for within-group IV(s)
                         Knowledge),
               type=3)
                                             # type of sum of squares (SS)
                                                                          Main effect of S
                                                                          No effect of K
                                             # output table of results
m2
                                                                          No S:K interaction
                                                            p p</.05/
                      Effect DFn DFd
                                                                              ges
            Stereotypicality 1 49 19.6218320 5.293689e-05
                                                                    10.0548009878
                              1 49 1.1620173 2.863259e-01
                                                                    0.0079323037
                   Knowledge
4 Stereotypicality: Knowledge 1 49 0.1098314 7.417495e-01
                                                                    0.0003880361
```

Calculate descriptive statistics using ezStats()

```
Cond N Mean SD FLSD

1 ms.mk 50 356.48 57.27493 19.69711

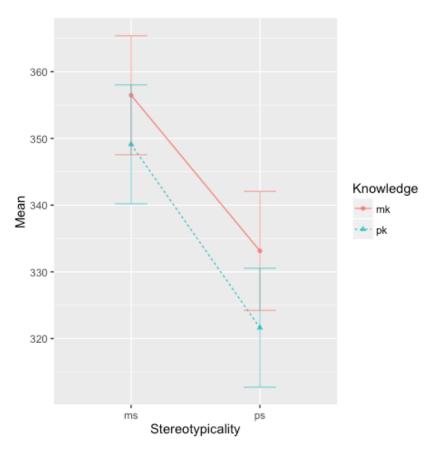
2 ms.pk 50 349.12 56.48248 19.69711

3 ps.mk 50 333.14 54.12835 19.69711

4 ps.pk 50 321.62 44.40762 19.69711
```

Plot results using ezPlot()

Interaction Plot



Reporting the Results

Report all of the following:

- The observed difference between conditions
- The specific kind of test (e.g., t-test)
- The computed statistic (e.g., t)
- Degrees of freedom for the test
- The p-value of the test
- The effect size (e.g., r²)

"The mean response times for critical words were fastest in the +s+k condition, slowest for the -s-k condition, and intermediate for the +s-k and -s-k conditions (see Table 1). A 2x2 repeated measures ANOVA revealed a significant main effect of stereotypicality, $F_1(1,49) = 19.62$, p < .001, GES = 0.05, $F_2(1,39) = 8.33$, p < .01, GES = 0.07. No effect of speaker knowledge (Fs < 2) or an interaction between stereotypicality or speaker knowledge (Fs < 2) were found."

INTERPRETTING THE RESULTS

Possible outcomes of a 2x2 design

Interpreting the Results

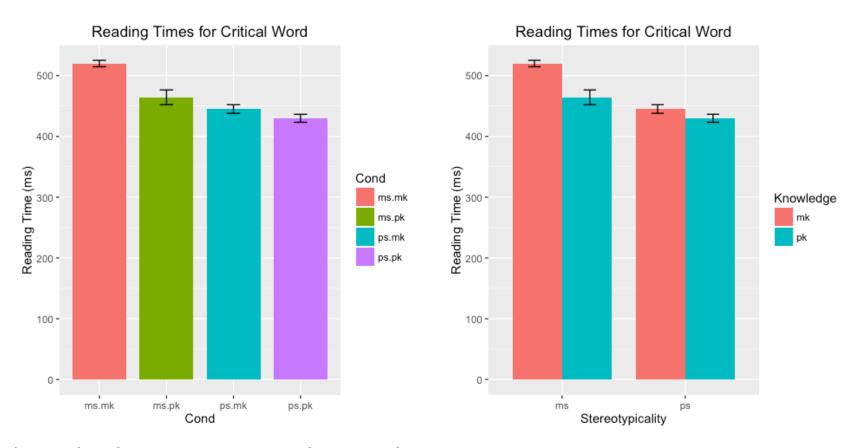
After collecting data, compare conditions to see whether IV(s) and/or their interaction had an effect on the DV

- **Graphs** useful for identifying patterns and summarizing results
- Statistical analyses tell us whether the results are likely to be "real"

Plotting the results (hypothetical data*)

Bar graph

- y-axis: DV, x-axis: IV levels
- Error bars usually represent Standard Error of the Mean (SEM))

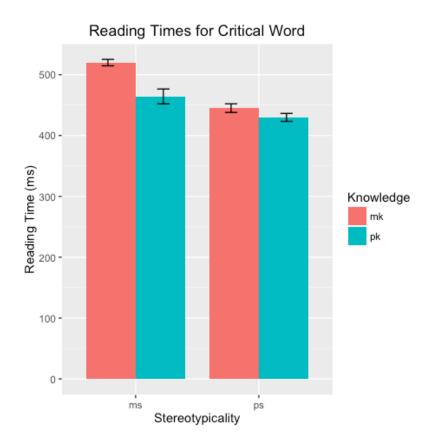


^{*} Each of the following slides uses a different set of hypothetical data

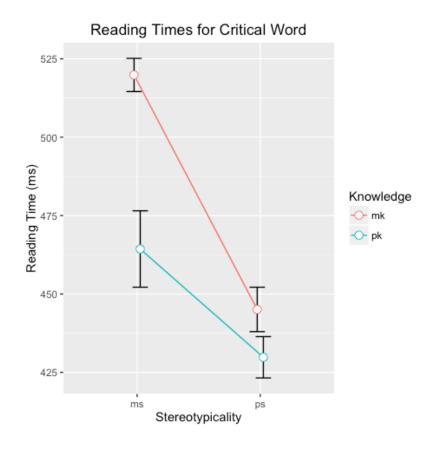
Plotting the results (hypothetical data)

Bar graph

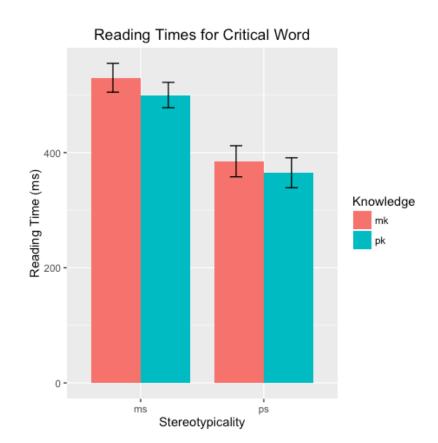
_ -

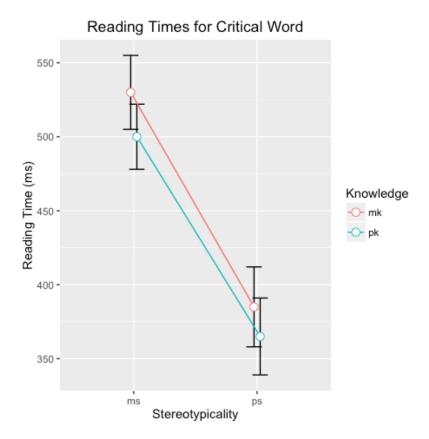


Line graph (Interaction plot)

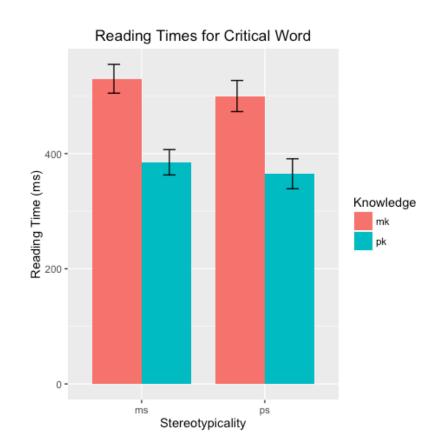


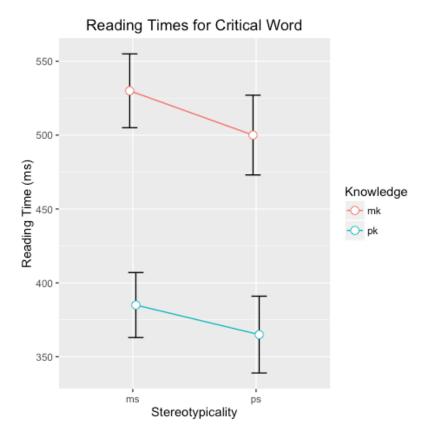
One main effect



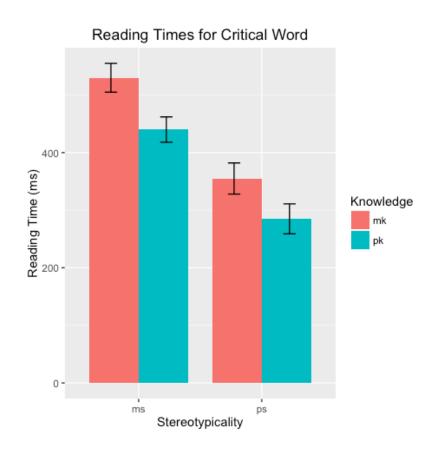


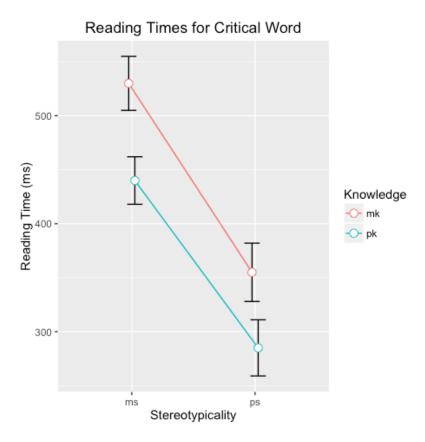
One main effect



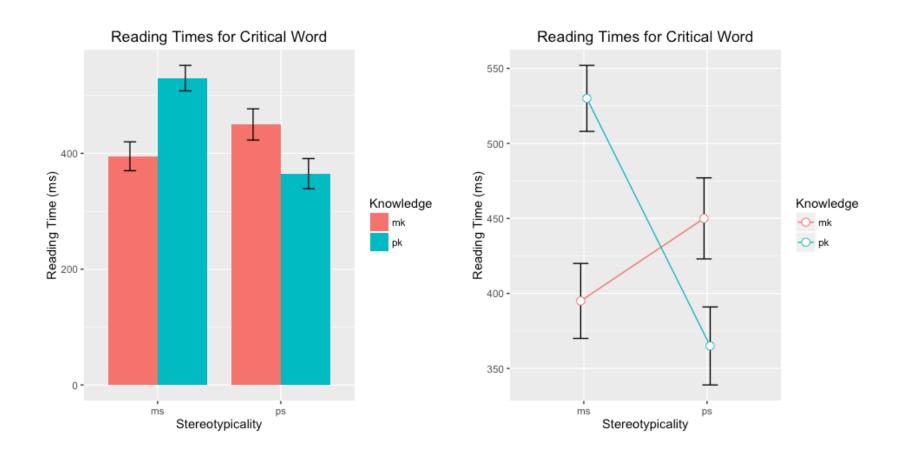


Two main effects

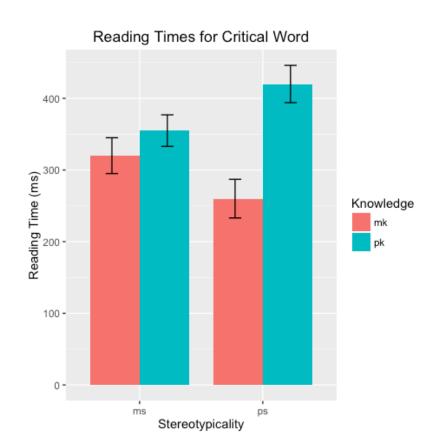


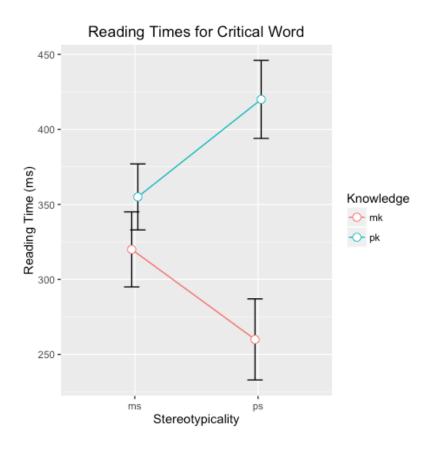


Interaction (but no main effects)



Interaction (but no main effects)





Plotting the results (hypothetical data)

Two main effects and an interaction



Knowledge

mk

pk

Interpreting the Results

Don't forget!

- **Graphs** useful for identifying patterns and summarizing results
- Statistical analyses tell us whether the results are likely to be "real"

Summary

Testing Hypotheses

- 1. State the null (H_0) and alternative hypotheses (H_1)
- 2. Set the decision criteria (e.g., choose alpha level)
- 3. Sample from a population (collect data)
- 4. Describe data and calculate appropriate test statistics
- 5. Make a decision (reject H_0 or fail to reject H_0)