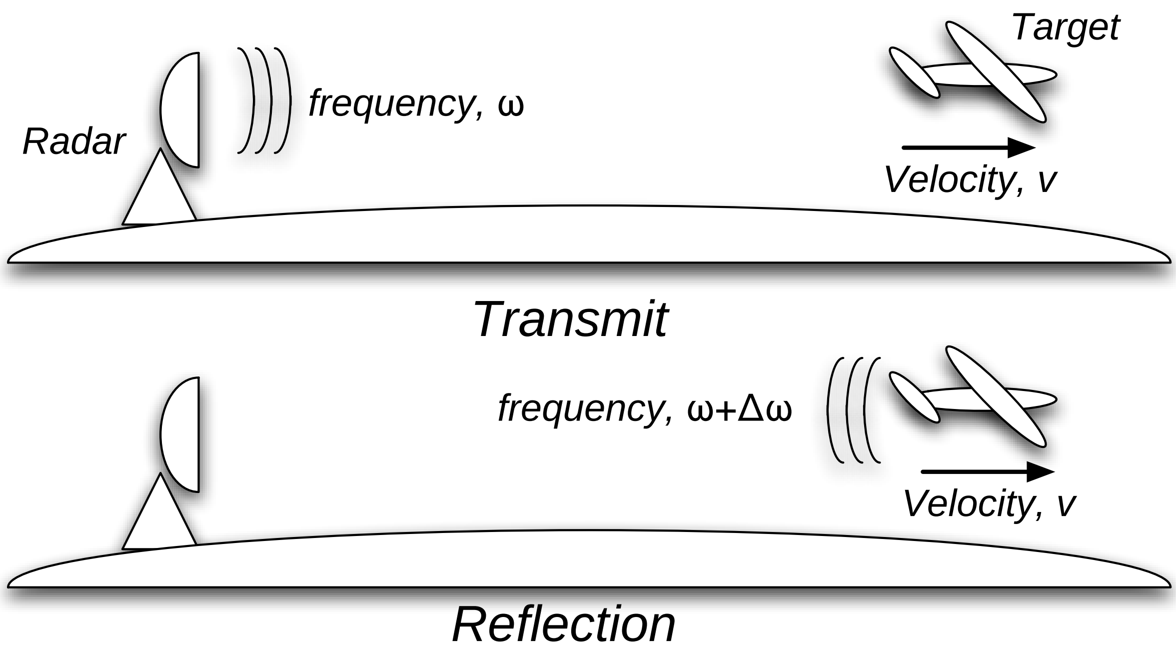
Signal Processing and Linear Systems I

Applications 4: Doppler Ultrasound

February 22, 2015

Basic Doppler Radar Idea



The target is moving.

The reflected RF pulse is shifted in frequency.

Doppler Shift

The target is a distance d from the antenna, moving at a velocity v.

The phase of the received signal is the total distance divided by the wavelength λ.

$$\phi = -2\pi \frac{2d}{\lambda}$$

The negative sign is due to the fact that the received signal is *delayed* in time.

The frequency of the signal is

$$\omega_d = \frac{d}{dt}\phi = \frac{d}{dt}\left(-2\pi\frac{2d}{\lambda}\right) = -2\pi\frac{2v}{\lambda}$$

The wavelength $\lambda = c/(\omega/2\pi) = 2\pi c/\omega_c$, where c is the speed of light, so

$$\omega_d = -2\pi\frac{2v}{2\pi c/\omega_c} = -\frac{2v}{c}\omega_c$$

or, dividing both sides by 2π, and using $f = \omega/2\pi$,

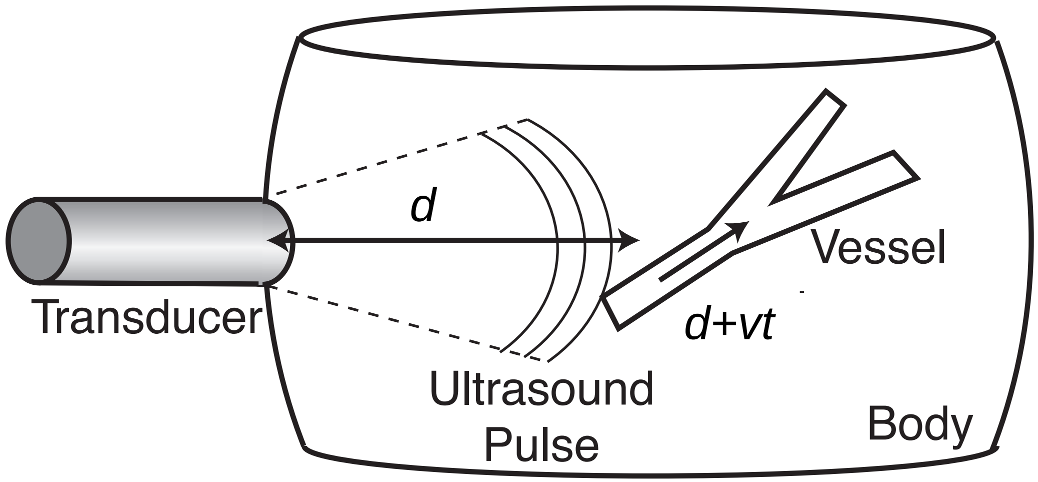
$$f_d = -\frac{2v}{c}f_c$$

For an airplane traveling 300 m/s (just below the speed of sound, 330 m/s), and a radar frequency of $f_c = 1$GHz, this gives

$$f_d = -\frac{(2)(3\times10^2\ \text{m/s})}{3\times10^8\ \text{m/s}}(1\times10^9\ \text{Hz}) = -2\times10^3\ \text{Hz} = -2\ \text{kHz}.$$

Ultrasound

Ultrasound works like a radar system:



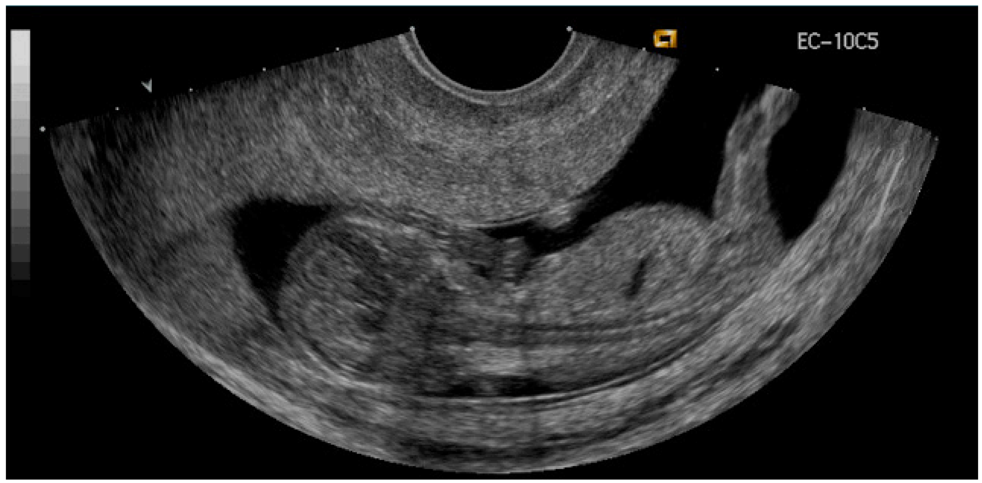
- The transducer produces a sound pulse
- The sound is reflected from the tissue at some depth
- The reflected sound is detected by the transducer
- The round trip time tells you how deep the tissue reflector is

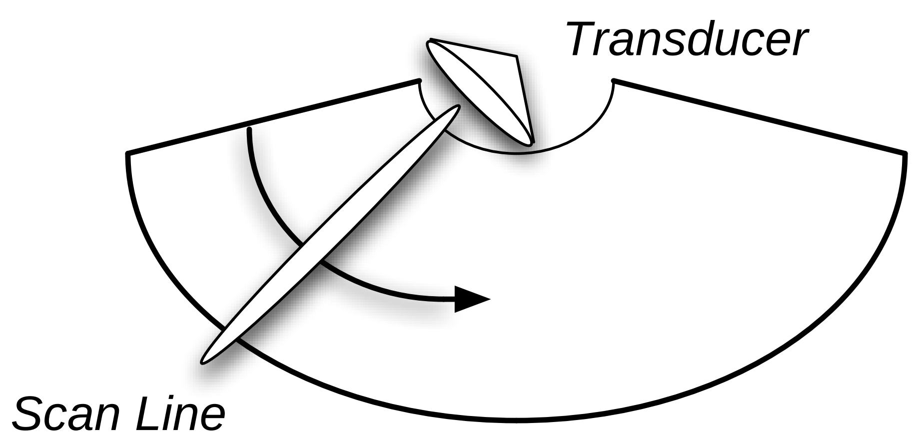
Ultrasound Scanners

Cart based system

Portable System

Ultrasound Imaging





The speed of sound in tissue is $c = 1500$ m/s, and typical carrier frequencies are 1 to 10 MHz.

To detect a voxel at a depth of 15 cm requires a round-trip time of

$$t_d = 2d/c = 2 \times 0.15 \text{ m}/1500 \text{ m/s} = 2 \times 10^{-4} \text{ s} = 200\ \mu\text{s}$$

Every $\sim 200\ \mu$s a new pulse is transmitted, and a line in the image acquired.

The beam is scanned (originally mechanically, now electrically), and the image painted every 100 ms or so.

Moving blood shows a Doppler shift. A typical velocity is 1 m/s, and $f_c = 3$ MHz

$$f_d = -\frac{2v}{c} f_c = -\frac{(2)(1 \text{ m/s})}{1500 \text{ m/s}} 3 \times 10^6 \text{ Hz} = -4 \text{ kHz}.$$

Frequency shifts are right in the audio range.

How long do we need to measure 4 kHz? To make it easy, we need one cycle (think of the fundamental frequency for the Fourier series), so

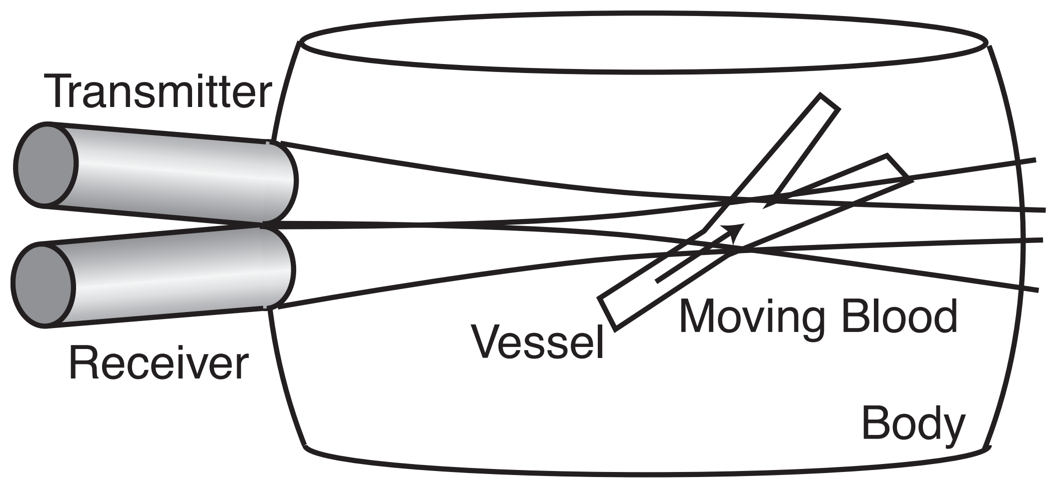
$$T = 1/(4 \text{ kHz}) = 0.25 \text{ ms} = 250\ \mu\text{s}$$

Ideally we'd like some multiple of this, so we could resolve many velocities.

Problem: This is longer than the round-trip time!

Continuous-Wave (CW) Doppler

Simple solution: give up on spatial resolution, and transmit all the time.



Transmit and receive transducers are separate, and overlap over the region of interest.

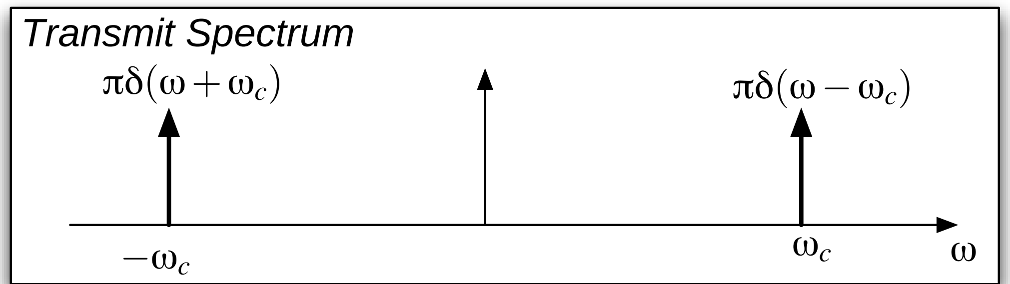
Typical of fetal heart monitors.

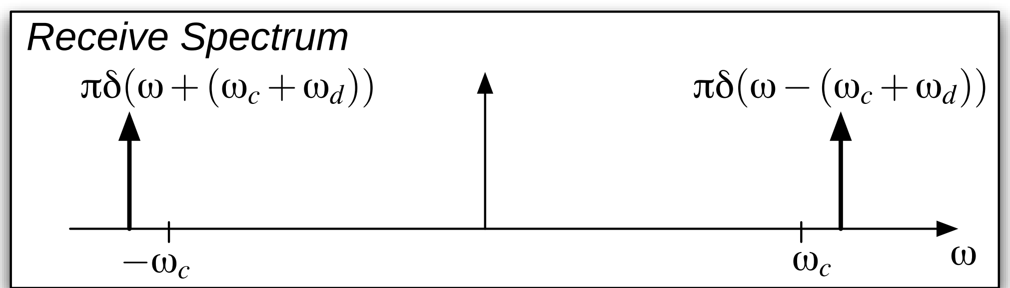
You can have one of your own!

Transmit and Receive Spectra

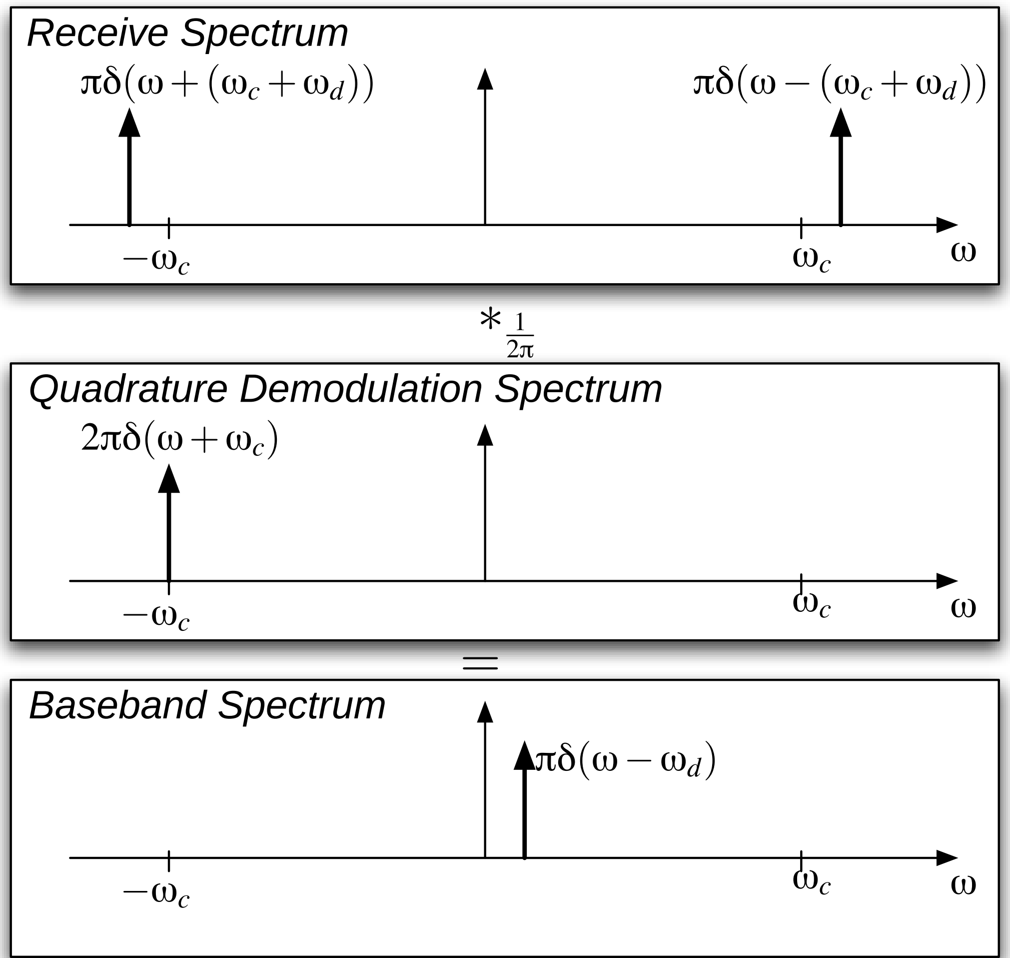
The transmit signal is $\cos(\omega_c t)$. If we ignore phase shifts, the received signal is the Doppler shifted $\cos((\omega_c + \omega_d)t)$.

The spectra are then





If we use a quadrature receiver demodulating with $e^{-j\omega_c t}$, this corresponds to convolving with $2\pi\delta(\omega + \omega_c)$ in the frequency domain



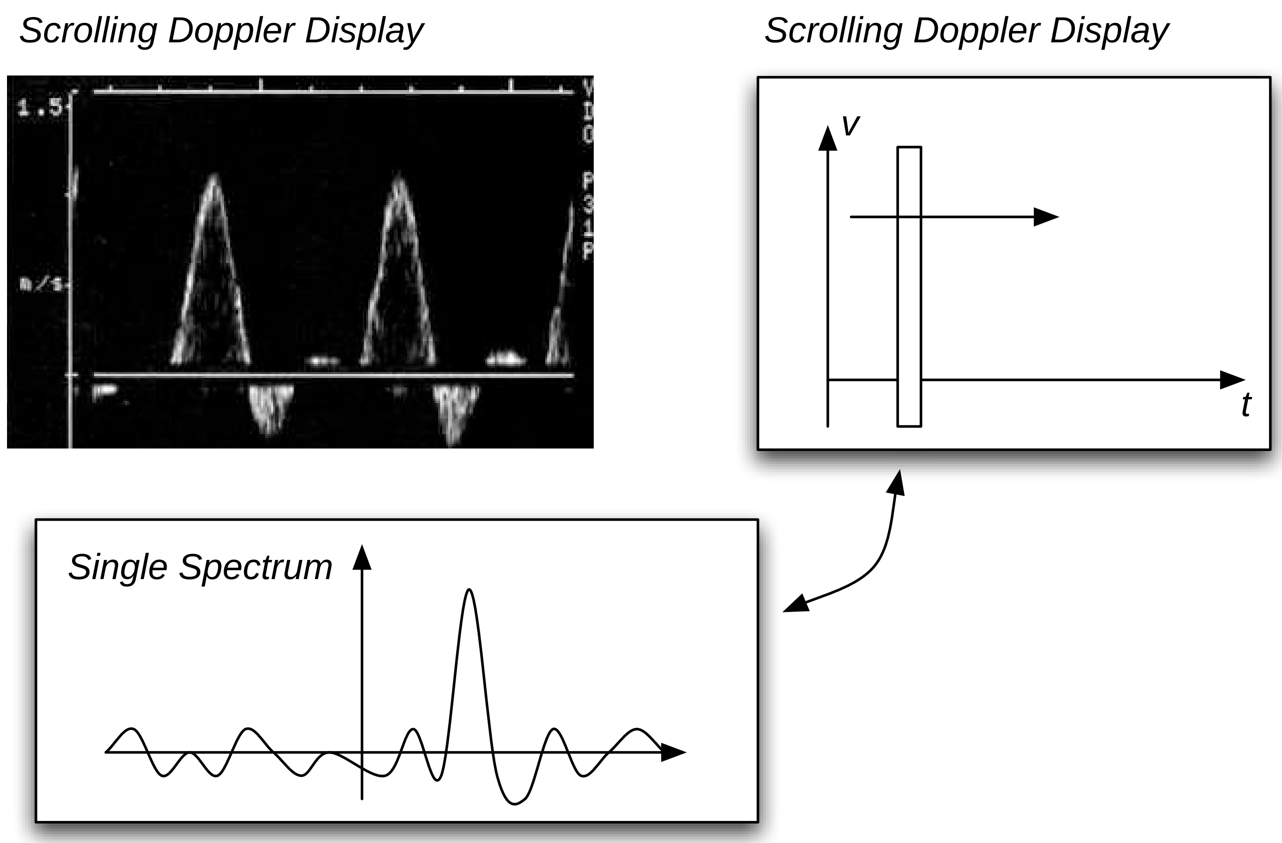
We are left with the baseband (Doppler) signal, plus a term near $-2\omega_c$ which we suppress with a filter.

Note that the baseband signal is *complex*.

The sign of the frequency tells you which way the blood is moving.

CW Doppler Display

Blood flow speed changes with time. This is shown with a scrolling display



Pulsed Doppler

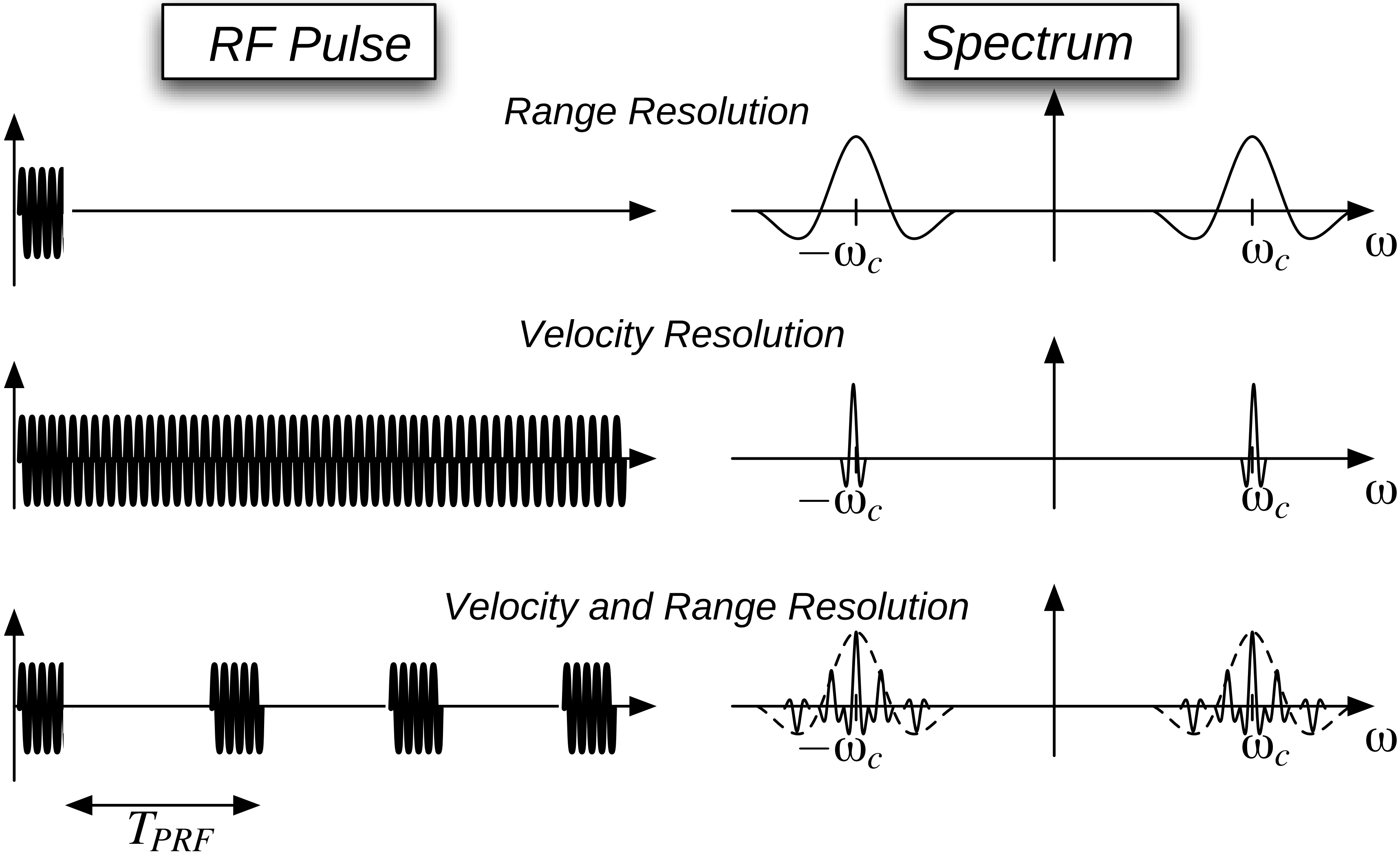
A single, short pulse gives good range resolution, but poor frequency (velocity) resolution.

A single, long pulse gives good frequency (velocity) resolution, but poor range resolution.

How do we get both, to localized the velocity in a particular vessel, or heart valve?

Many different solutions.

A common solution in ultrasound and radar is *Pulsed Doppler*, shown on the next page.



Pulsed Doppler waveform

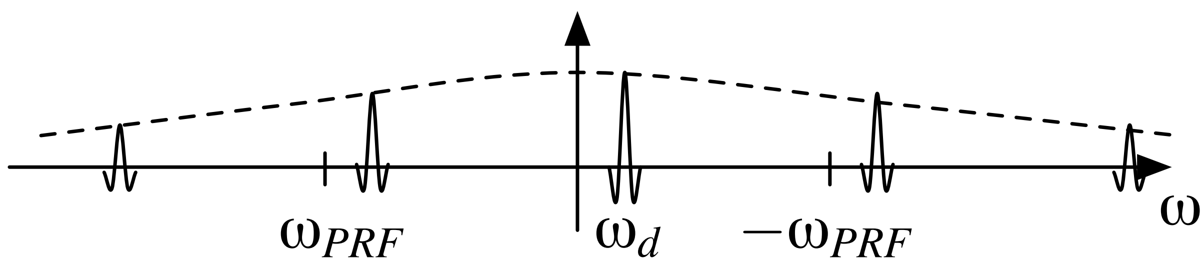
A short pulse is repeated many times, at an spacing T_{PRF}.

The round-trip time must be less than this

$$t_d = \frac{2d}{c} < T_{PRF}$$

Otherwise a closer voxel will also be returning signal at the same time, and the signal is *ambiguous*.

After quadrature demodulation, the baseband spectrum looks like



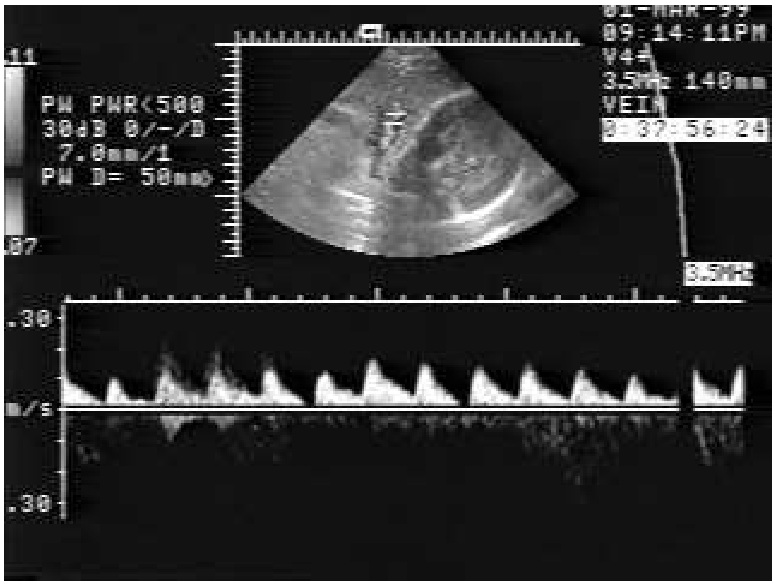
The spacing between the sublobes is $f_{PRF} = 1/T_{PRF}$, and this must be greater than the Doppler shift

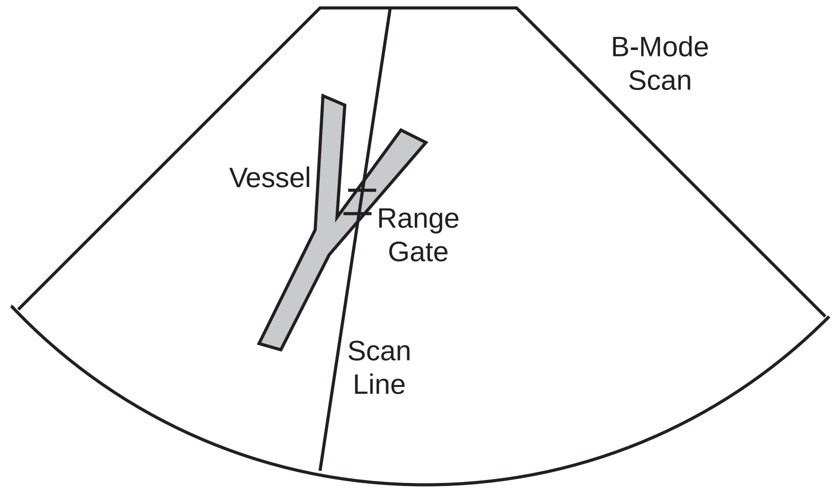
$$f_d = \frac{2v}{c} f_c < f_{PRF}$$

or the velocity is ambiguous. This is the same as sampling.

Often, f_{PRF} is not high enough due to the round-trip time required to reach a specific depth, d, and aliasing occurs.

Pulsed Doppler Ultrasound

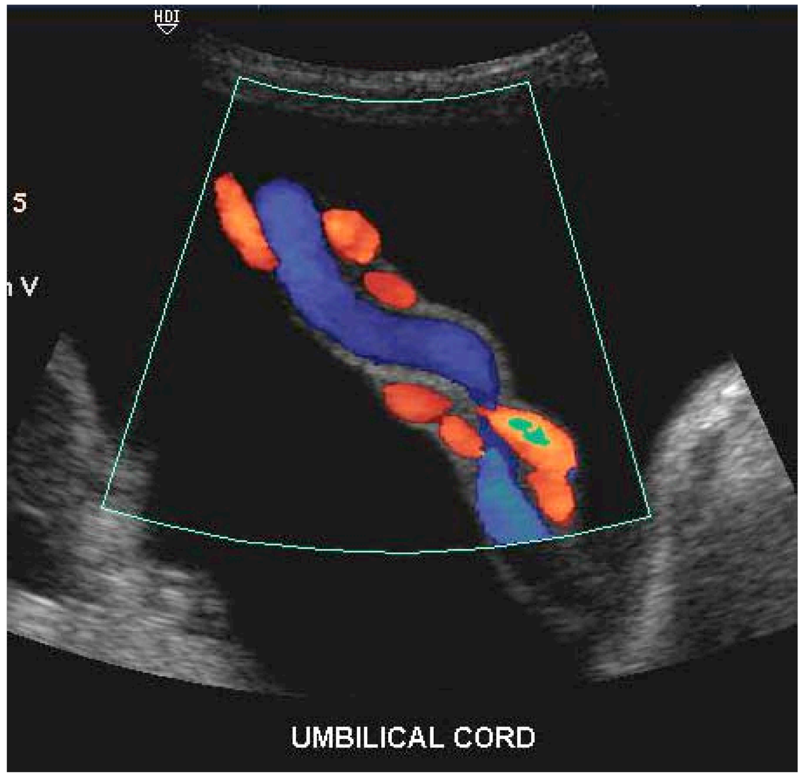




A range gate is positioned on the ultrasound image (B-mode scan). The depth sets T_{PRF}.

A pulsed Doppler waveform interrogates the velocities in that voxel, which appears on the scrolling display.

Color Flow Ultrasound



Encode the magnitude and direction of the flow in color.

Superimpose this on a grayscale image.