EE 301 Lab 2 – Convolution

1 Introduction

In this lab we will gain some more experience with the convolution integral and create a script that shows the graphical method of convolution.

2 What you will learn

This lab will focus on using MATLAB® to graphically portray convolution. You will then verify the effect of pulse width spreading and output interference.

In the second part of this lab you will calculate the impulse response of an RC circuit. You will then compare your results with an in-lab exercise.

3 Background Information and Notes

None.

4 Guided Exercises

a. **Convolution:**
Reproduce the following m-file script and run the program.

```
%% Script M-file graphically demonstrates the convolution process
%% By B. P. Lathi, Linear Systems and Signals, page 232, ex M2.4

%% Create figure window and make visible on screen
figure(1)

x = inline('1.5*sin(pi*t).*(t>=0 & t<1)');
h = inline('1.5*(t>=0 & t<1.5)-(t>=2 & t<2.5)');
dtau = 0.005;
tau = -1:dtau:4;
ti = 0;
tvec = -1:0.1:4;

%% Pre-allocate memory
y = NaN*zeros(1,length(tvec));
for t = tvec
  %% Time index
  ti = ti+1;
  xh = x(t-tau).*h(tau);
  lxh = length(xh);
  %% trapezoidal approximation of integral
  y(ti) = sum(xh.*dtau);

  subplot(2,1,1), plot(tau,h(tau), 'r-', tau, x(t-tau), 'b--',t,0,'ok');
```

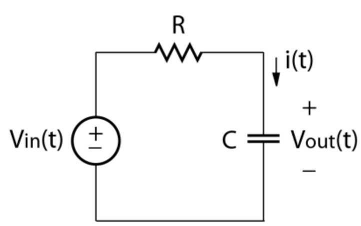
```
    axis([tau(1) tau(end) -2.0 2.5]);
    %% patch command is used to create the gray-shaded area of convolution
    patch([tau(1:end-1); tau(1:end-1); tau(2:end); tau(2:end)],...
      [zeros(1,lxh-1); xh(1:end-1); xh(2:end); zeros(1,lxh-1)],...
      [0.8 0.8 0.8], 'edgecolor','none');
    xlabel('\tau');
    legend('h(\tau)', 'x(t-\tau)','t','h(\tau)x(t-\tau)',3);

    c = get(gca, 'children');
    set(gca,'children', [c(2);c(3);c(4);c(1)]);

    subplot(2,1,2), plot(tvec,y,'k',tvec(ti),y(ti),'ok');
    xlabel('t');
    ylabel('y(t)');
    title('y(t) = \int h(\tau)x(t-\tau) d\tau');
    axis([tau(1) tau(end) -1.0 2.0]);
    grid;

    %% drawnow command updates graphics windoe for each loop iteration
    drawnow;

    %% Using the pause command allows one to manually step through the
    %% convolution process
    % pause;

end
```

Q 1. Exercising graphical convolution: At the end of the program, take a screen shot and include this in your lab report.

Q 2. Comparing with HW2 Prob4: Modify the program to graphically show your result of EE301 HW2 Problem 4. Provide two versions; one with the appropriate pulse spacing and one without the appropriate pulse spacing. You may just use two pulses instead of a train of pulses and you may modify the location of the pulses.

b. RC circuit

The RC circuit shown below is an example of an LTI system.



The system input is the voltage $V_{in}(t)$ and the system output is the voltage across the capacitor, $V_{out}(t)$.

Q 3. 1st order differential equation: With the use of Kirchoff's voltage law and the relationship between current and voltage in a capacitor, show that:

$$RC\frac{dV_{out}(t)}{dt}+V_{out}(t)=V_{in}(t).$$

The solution to the above equation is:

$$V_{out}(t)=V_0 e^{-t/(RC)}+\int_0^t \frac{1}{RC}e^{-(t-\tau)/(RC)}V_{in}(\tau)d\tau, \qquad t\geq 0,$$

where we will assume the initial condition of the voltage across the capacitor is zero, or V_0 = 0 volts.

c. Impulse Response of RC circuit

To obtain the impulse response of the RC circuit, we must supply an impulse function (delta function) as the input. However, a delta function is not a physically realizable signal, and even if it were available, we may fry or damage the circuit. We must then resort to the Heaviside unit step function to help us obtain the impulse response. Remember that we can use two unit step functions to create a rectangular pulse signal. This rectangular pulse signal can be used to approximate the delta function as the width of the pulse becomes sufficiently small. Let us define the pulse as,

$$p_\Delta(t)=\frac{1}{\Delta}\left[u(t)-u(t-\Delta)\right] \qquad \text{for } \Delta>0,$$

d. Step Response of RC circuit

The convolution integral allows us to obtain the system response, y(t), if we know the system's impulse response, h(t), and the input signal, x(t). The output is then,

$$y(t)=x(t)*h(t)=\int_{-\infty}^{+\infty}x(\tau)h(t-\tau)d\tau=\int_{-\infty}^{+\infty}x(t-\tau)h(\tau)d\tau,$$

where the symbol * denotes the convolution operation.

The step response, $y_s(t)$, is found by applying a Heaviside step function as the input, or x(t) = u(t). Thus,

$$y_s(t)=u(t)*h(t)=\int_{-\infty}^{t}h(\tau)\,u(t-\tau)d\tau=\int_{-\infty}^{t}h(\tau)\,d\tau,$$

since the Heaviside step function evaluates to 1 for t - τ > 0 and 0 for t - τ < 0.

By taking the derivative with respect to time of $y_s(t)$ we find that,

$$\frac{dy_s(t)}{dt}=\frac{d}{dt}\int_{-\infty}^{t}h(\tau)\,d\tau=h(t).$$

The above result tells us that the impulse response h(t) can be computed by calculating the derivative of the step response with respect to time.

Q 4. Obtain $y_s(t)$: Using the result of the 1st order differential equation that relates $V_{in}(t)$ to $V_{out}(t)$, obtain the step response $y_s(t) = V_{out}(t)$ if the input is a Heaviside step function, $u(t) = V_{in}(t)$.
Aside: In some textbooks the Heaviside step function is denoted as H(t) not u(t).

Q 5. Obtain h(t): Using the step response, $y_s(t)$, find the impulse response h(t).

Q 6. Obtain pulse response part 1: Using the pulse p_Δ (as defined above) as the input $V_{in}(t)$, and using the impulse response h(t) that you found in Q 5, determine the output response $V_{out}(t)$. We will call this output response $y_\Delta(t)$.

Q 7. Obtain pulse response part 2: Compute $\lim_{\Delta \to 0} y_\Delta(t)$. You can use l'Hopital's rule to evaluate $\lim_{\Delta \to 0} \frac{e^{\Delta/(RC)} - 1}{\Delta}$.

e. ☆ **EXTRA CREDIT** ☆

In-Lab Verification of Pulse Response:
Build the RC circuit on a protoboard, with R = 1 kΩ and C = 1 μF. You may need to be flexible on the values of R and C. As a reference, you can plot the expected capacitor voltage as a function of time using Matlab.

Connect Channel 1 to measure $V_{in}(t)$ and Channel 2 to measure $V_{out}(t)$. Make sure that the oscilloscope channels are set for DC coupling.

Use the function generator to produce a 100 mV pulse with duration of 1 ms. Collect the measured data. Repeat for a 200 mV pulse with duration 0.5 ms and again for a 1 V pulse with duration 0.1 ms.

Leaving the duty cycle of the square wave to 50%, what happens as you decrease the duration of the pulse? In terms of the convolution process, what do you expect is happening?

Can you devise a plan to obtain the impulse response? Hint: It is important to consider the the time constant of the RC circuit.

5 Review

1. None.

6 Lab Report

1. The first page of your Lab report should be a cover sheet with your name, USC ID and Lab #. Please note that all reports should be typed.
2. Answer all the questions which were asked in the lab report. Kindly display the code lines you executed to arrive at your answer along with figures to support them. Please give written explanation or put comment lines where necessary. Please note that each figure should have proper labels for the x and y axis and should have a suitable title.
3. Answer the review questions.
4. Submit a printout of your completed M-file documenting all the lab exercises.