

Table 3.1: Properties of Continuous Time Fourier Series

Property	Periodic Signal	Fourier Transform
$\begin{cases} x(t) \\ y(t) \end{cases}$ <i>Periodic with Period T and</i> <i>fundamental frequency $\omega_0 = \frac{2\pi}{T}$</i>		$\begin{cases} a_k \\ b_k \end{cases}$
Linearity	$Ax(t) + By(t)$	$Aa_k + Bb_k$
Time Shifting	$x(t - t_0)$	$a_k e^{-jk\omega_0 t_0} = a_k e^{-jk(\frac{2\pi}{T})t_0}$
Frequency Shifting	$e^{jM\omega_0 t} x(t)$	a_{k-M}
Conjugation	$x^*(t)$	a_{-k}^*
Time Reversal	$x(-t)$	a_{-k}
Time Scaling	$x(at), \quad a > 0$ <i>(Periodic with period $\frac{T}{a}$)</i>	a_k
Periodic Convolution	$x(t) * y(t) = \int_T x(\tau)y(t - \tau)d\tau$	$Ta_k b_k$
Multiplication	$x(t)y(t)$	$\sum_{l=-\infty}^{+\infty} a_l b_{k-l}$
Differentiation	$\frac{d}{dt}x(t)$	$jk\omega_0 a_k = jk\frac{2\pi}{T} a_k$
Integration	$\int_{-\infty}^t x(\tau)d\tau$ (finite value and periodic only if $a_0 = 0$)	$\left(\frac{1}{jk\omega_0}\right)a_k = \left(\frac{1}{jk\frac{2\pi}{T}}\right)a_k$
Conjugate Symmetry for Real Signals	$x(t)$ is real	$\begin{cases} a_k = a_{-k}^* \\ \text{Re}\{a_k\} = \text{Re}\{a_{-k}\} \\ \text{Im}\{a_k\} = -\text{Im}\{a_{-k}\} \\ a_k = a_{-k} \\ \angle a_k = -\angle a_{-k} \end{cases}$
Symmetry for Real and Even Signals	$x(t)$ is real and even	a_k is real and even
Symmetry for Real and Odd Signals	$x(t)$ is real and odd	a_k is purely imaginary and odd
Even-Odd Decomposition for Real Signals	$x_r(t) = \text{Ev}\{x(t)\} \quad [x(t)\text{real}]$ $x_o(t) = \text{Od}\{x(t)\} \quad [x(t)\text{real}]$	$\text{Re}\{a_k\}$ $j\text{Im}\{a_k\}$
Parseval's Relation for Periodic Signals	$\frac{1}{T} \int_T x(t) ^2 dt = \sum_{k=-\infty}^{\infty} a_k ^2$	

Table 3.2: Properties of Discrete Time Fourier Series

Property	Periodic Signal	Fourier Transform
$\begin{cases} x[n] \\ y[n] \end{cases}$ <i>Periodic with Period N and fundamental frequency $\omega_0 = \frac{2\pi}{N}$</i>		$\begin{cases} a_k \\ b_k \end{cases}$ <i>Periodic with Period N</i>
Linearity	$Ax[n] + By[n]$	$Aa_k + Bb_k$
Time Shifting	$x[n - n_0]$	$a_k e^{-jk(\frac{2\pi}{N})n_0}$
Frequency Shifting	$e^{jM(\frac{2\pi}{N})n} x[n]$	a_{k-M}
Conjugation	$x^*[n]$	a_{-k}^*
Time Reversal	$x[-n]$	a_{-k}
Time Scaling	$x_{(m)}[n]$ $= \begin{cases} x[\frac{n}{m}] & \text{if } n \text{ is a multiple of } m \\ 0 & \text{if } n \text{ is not a multiple of } m \end{cases}$ (periodic with period mN)	$\frac{1}{m} a_k$ (viewed as periodic With period mN)
Periodic Convolution	$\sum_{r=\langle N \rangle} x[r]y[n-r]$	$Na_k b_k$
Multiplication	$x[n]y[n]$	$\sum_{l=\langle N \rangle} a_l b_{k-l}$
First Difference	$x[n] - x[n-1]$	$(1 - e^{-jk2\pi/N})a_k$
Running Sum	$\sum_{k=-\infty}^n x[k]$ Finite valued and periodic only if $a_0 = 0$	$\left(\frac{1}{\left(1 - e^{-\frac{jk2\pi}{N}}\right)} \right) a_k$
Conjugate Symmetry for Real Signals	$x[n]$ is real	$\begin{cases} a_k = a_{-k}^* \\ \text{Re}\{a_k\} = \text{Re}\{a_{-k}\} \\ \text{Im}\{a_k\} = -\text{Im}\{a_{-k}\} \\ a_k = a_{-k} \\ \angle a_k = -\angle a_{-k} \end{cases}$
Symmetry for Real and Even Signals	$x[n]$ is real and even	a_k is real and even
Symmetry for Real and Odd Signals	$x[n]$ is real and odd	a_k is purely imaginary and odd
Even-Odd Decomposition for Real Signals	$x_r[n] = \text{Ev}\{x[n]\}$ $[x(t)\text{real}]$ $x_o[n] = \text{Od}\{x[n]\}$ $[x(t)\text{real}]$	$\text{Re}\{a_k\}$ $j\text{Im}\{a_k\}$
Parseval's Relation for Periodic Signals	$\frac{1}{N} \sum_{n=\langle N \rangle} x[n] ^2 = \sum_{k=\langle N \rangle} a_k ^2$	

Table 4.1: Properties of the Fourier Transform		
Property	Aperiodic Signal	Fourier Transform
	$x(t)$ $y(t)$	$X(j\omega)$ $Y(j\omega)$
Linearity	$ax(t) + by(t)$	$aX(j\omega) + bY(j\omega)$
Time Shifting	$x(t - t_0)$	$e^{-j\omega t_0}X(j\omega)$
Frequency Shifting	$e^{j\omega_0 t}x(t)$	$X(j(\omega - \omega_0))$
Conjugation	$x^*(t)$	$X^*(-j\omega)$
Time Reversal	$x(-t)$	$X(-j\omega)$
Time and Frequency Scaling	$x(at)$	$\frac{1}{ a }X\left(\frac{j\omega}{a}\right)$
Convolution	$x(t) * y(t)$	$X(j\omega)Y(j\omega)$
Multiplication	$x(t)y(t)$	$\frac{1}{2\pi}X(j\omega) * Y(j\omega)$
Differentiation in Time	$\frac{d}{dt}x(t)$	$j\omega X(j\omega)$
Integration	$\int_{-\infty}^t x(\tau)d\tau$	$\frac{1}{j\omega}X(j\omega) + \pi X(0)\delta(\omega)$
Differentiation in Frequency	$tx(t)$	$j\frac{d}{d\omega}X(j\omega)$
Conjugate Symmetry for Real Signals	$x(t) \text{ is real}$	$\begin{cases} X(j\omega) = X^*(-j\omega) \\ \text{Re}\{X(j\omega)\} = \text{Re}\{X(-j\omega)\} \\ \text{Im}\{X(j\omega)\} = -\text{Im}\{X(-j\omega)\} \\ X(j\omega) = X(-j\omega) \\ \Re X(j\omega) = -\Re X(-j\omega) \end{cases}$
Symmetry for Real and Even Signals	$x(t) \text{ is real and even}$	$X(j\omega) \text{ is real and even}$
Symmetry for Real and Odd Signals	$x(t) \text{ is real and odd}$	$X(j\omega) \text{ is purely imaginary and odd}$
Even-Odd Decomposition for Real Signals	$x_r(t) = \text{Ev}\{x(t)\} \quad [x(t) \text{ real}]$ $x_o(t) = \text{Od}\{x(t)\} \quad [x(t) \text{ real}]$	$\text{Re}\{X(j\omega)\}$ $j\text{Im}\{X(j\omega)\}$
Parseval's Relation for Aperiodic Signals	$\int_{-\infty}^{+\infty} x(t) ^2 dt = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(j\omega) ^2 d\omega$	

Table 4.2: Basic Fourier Transform Pairs

Signal	Fourier transform	Fourier Series Coefficients (if periodic)
$\sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$	$2\pi \sum_{k=-\infty}^{\infty} a_k \delta(\omega - k\omega_0)$	a_k
$e^{j\omega_0 t}$	$2\pi \delta(\omega - \omega_0)$	$a_1 = 1$ $a_k = 0$ otherwise
$\cos(\omega_0 t)$	$\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$	$a_1 = a_{-1} = \frac{1}{2}$ $a_k = 0$ otherwise
$\sin(\omega_0 t)$	$\frac{\pi}{j}[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$	$a_1 = -a_{-1} = \frac{1}{2j}$ $a_k = 0$ otherwise
$x(t) = 1$	$2\pi \delta(\omega)$	$a_1 = 1, a_k = 0, k \neq 0$ (this is the Fourier series representation for any choice of $T > 0$)
$x(t) = \begin{cases} 1, & t < T_1 \\ 0, & T_1 < t \leq \frac{T}{2} \end{cases}$ And $x(t) = x(t + T)$	$\sum_{k=-\infty}^{\infty} \frac{2 \sin(k\omega_0 T_1)}{k} \delta(\omega - k\omega_0)$	$\frac{\omega_0 T_1}{\pi} \text{sinc}\left(\frac{k\omega_0 T_1}{\pi}\right) = \frac{\sin(k\omega_0 T_1)}{k\pi}$
$\sum_{n=-\infty}^{\infty} \delta(t - nT)$	$\frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta\left(\omega - \frac{2\pi}{T}k\right)$	$a_k = \frac{1}{T}$ for all k
$x(t) = \begin{cases} 1, & t < T_1 \\ 0, & T_1 < t \end{cases}$	$\frac{2 \sin(\omega T_1)}{\omega}$	-----
$\frac{\sin(Wt)}{\pi t}$	$X(j\omega) = \begin{cases} 1, & \omega < W \\ 0, & W < \omega \end{cases}$	-----
$\delta(t)$	1	-----
$u(t)$	$\frac{1}{j\omega} + \pi \delta(\omega)$	-----
$\delta(t - t_0)$	$e^{-j\omega t_0}$	-----
$e^{-\alpha t} u(t), \text{Re}\{\alpha\} > 0$	$\frac{1}{\alpha + j\omega}$	-----
$t e^{-\alpha t} u(t), \text{Re}\{\alpha\} > 0$	$\frac{1}{(\alpha + j\omega)^2}$	-----
$\frac{t^{(n-1)}}{(n-1)!} e^{-\alpha t} u(t), \text{Re}\{\alpha\} > 0$	$\frac{1}{(\alpha + j\omega)^n}$	-----