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Time Frequency Characteristics

$X(j\omega) = |X(j\omega)|e^{\angle jX(j\omega)}$

$x(t) \rightarrow h(t) \rightarrow y(t) = x(t)*h(t)$

$X(j\omega)H(j\omega) = Y(j\omega)$

$|Y(j\omega)| = |X(j\omega)||H(j\omega)|$

$\angle Y(j\omega) = \angle X(j\omega) + \angle H(j\omega)$

Changing the magnitude of the signal leads to amplification or suppression.

Altering the phase of the signal makes it unrecognizable.

Linear Phase

$*\delta(t-t_0) \leftrightarrow \cdot e^{-j\omega t0}$

Information content at different frequencies is shifted by different amounts.

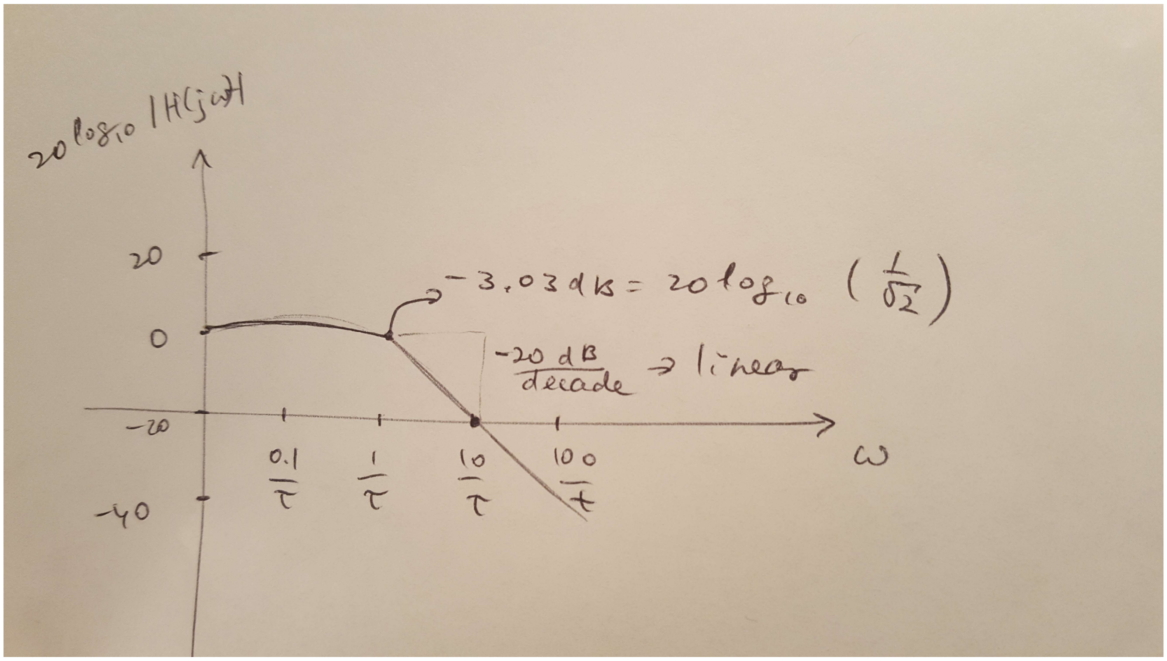
The shift in time by t_0 is a function of frequency and leads to "group delay."

Group Delay

$\alpha(\omega) = -\frac{d}{d\omega}\angle H(j\omega)$

Bode Plots

Bode plots are standard ways of drawing magnitude and phase response. The ω-axis is on a logarithmic scale. Bode plots typically deal with real valued responses. Because of this, they only show the positive ω-axis, since the negative one can be inferred by symmetry (same magnitude and opposite phase).



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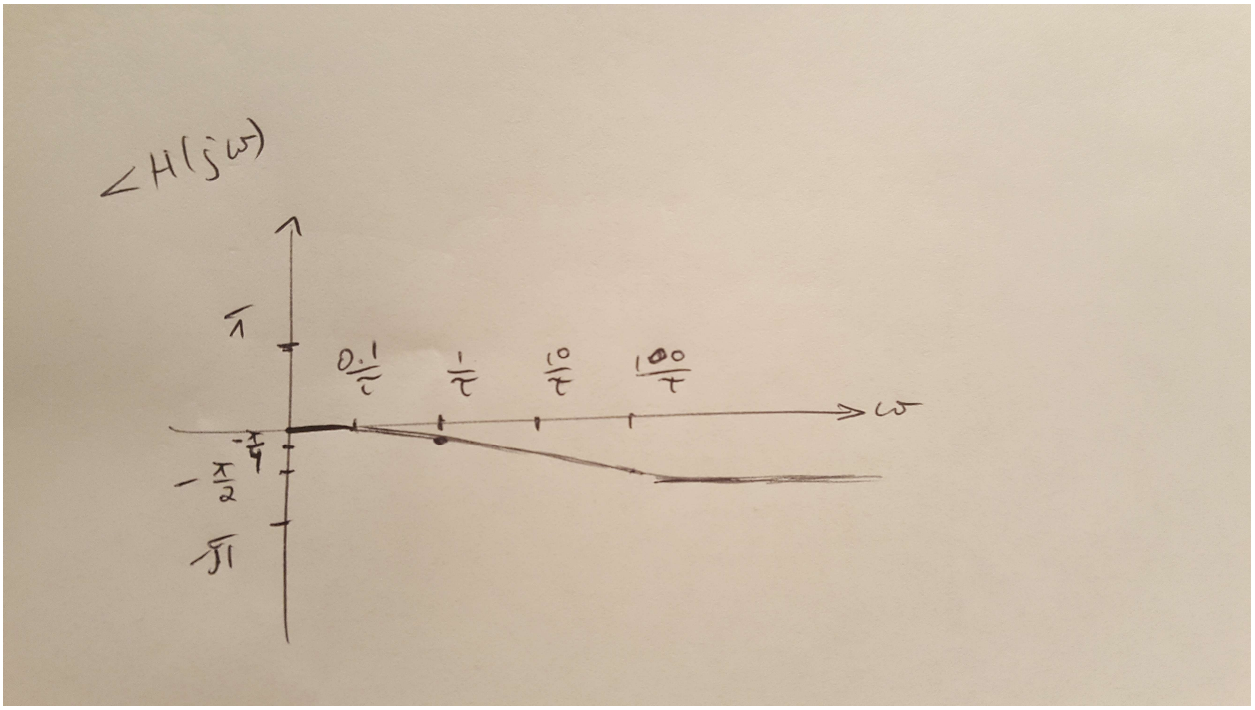
If 20log|H(jω)|>0 → The signal is amplified

If 20log|H(jω)|<0 → The signal is attenuated



First Order System

$\tau\frac{dy}{dt} + y(t) = x(t)$

$\tau j\omega Y(j\omega) + Y(j\omega) = X(j\omega)$

$(1+ \tau j\omega)Y(j\omega) = X(j\omega)$

$H(j\omega)=\frac{Y(j\omega)}{X(j\omega)} = \frac{1}{1+ \tau j\omega}$

At ω=0, H(jω)=1

20log1=0

$\omega=\frac{1}{\tau}$

$H(j\omega)= \frac{1}{1+ \tau j\frac{1}{\tau}} = \frac{1}{1+j}$

$$\left|\frac{1}{1+j}\right| = \frac{1}{\sqrt{2}}$$

$\omega=\frac{10}{\tau}$

H(jω)= $\frac{1}{1+10}$ → the imaginary term dominates

H(jω)≈$\frac{1}{10}$

20log(1/10)= - 20

ω=$\frac{100}{\tau}$

H(jω)= $\frac{1}{1+100}$ → the imaginary term dominates

H(jω)≈$\frac{1}{100}$

At ω=$\frac{1}{\tau}$

∠H(jω)=$-\frac{\pi}{4}$

If τ is big, ∠H(jω)≈ $-\frac{\pi}{2}$

For 2nd, 3rd order systems perform a partial fraction expansion to get in the form of:

$$\left(\frac{1}{1+\ \tau 1 j\omega}\right)\left(\frac{1}{1+\ \tau 2 j\omega}\right)\ldots$$

Multiplying magnitudes and adding phase.