Lecture 24:

Rotational Kinematics and Torque

Rotational physics makes me dizzy

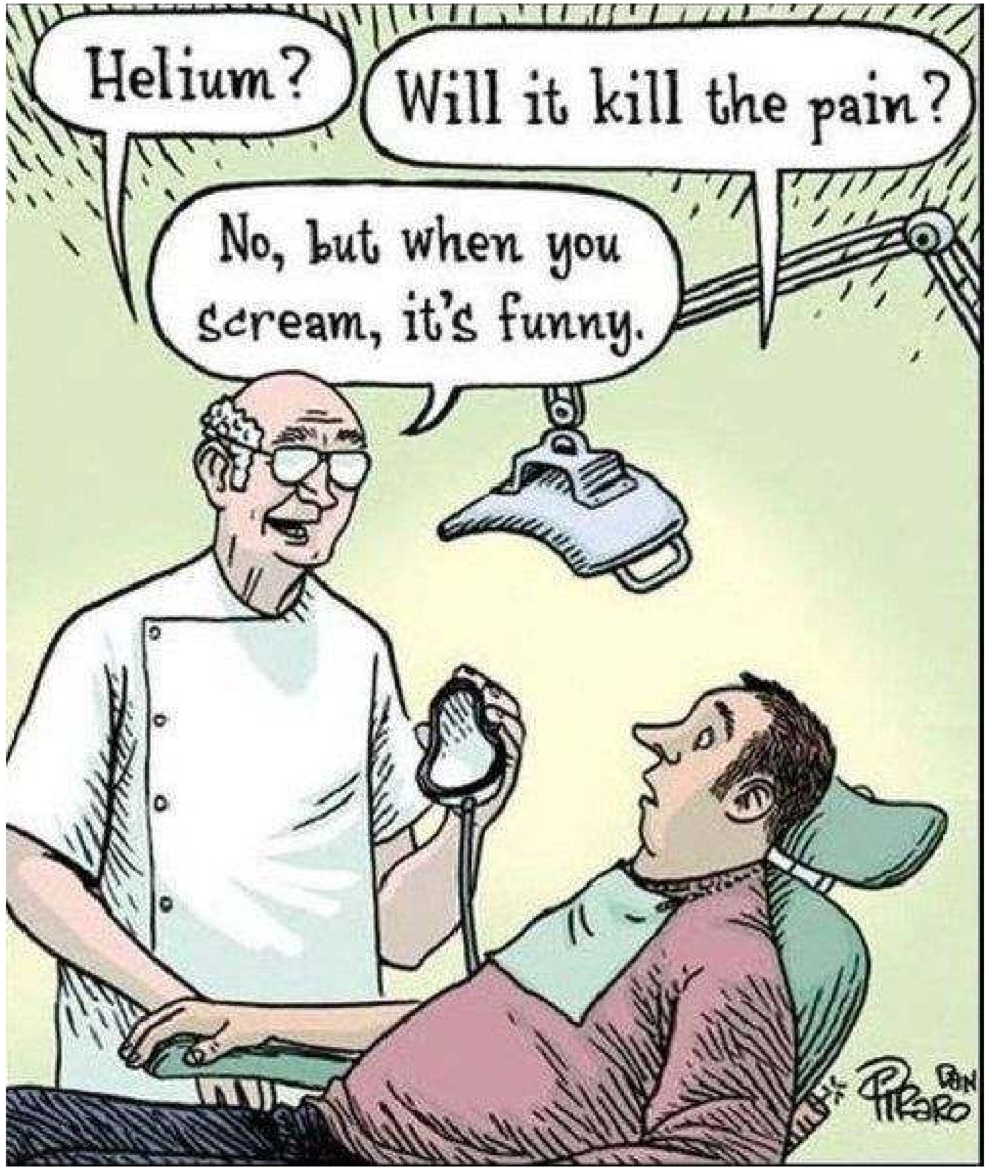
Erik Lascaris

Announcements

- HW 8 (gravity) was due yesterday!
  - Let me know if you need extension
- HW 9 = paper homework!
  - Available on piazza under "resources" tab!
  - Due next week: Tuesday 28 March
  - Please put your homework #9 into the HW box next to room SCI 121
- I have updated Lab #5 grades on WebAssign
  - Tuesday snow day labs have been excused
  - Email me if you missed the lab because of the snow but still have a zero
- Please check your clicker participation points on WebAssign
  - if you got a zero for a lecture you attended, email me to have it fixed

Root canal (part 2) .. the saga continues!

- No lecture on Monday April 3rd
- Because my apt. is at 12:30pm
- That's the Monday before the midterm!
  - Midterm is Apr 5th
- I should be back for office hours (I hope)



Today's Concepts

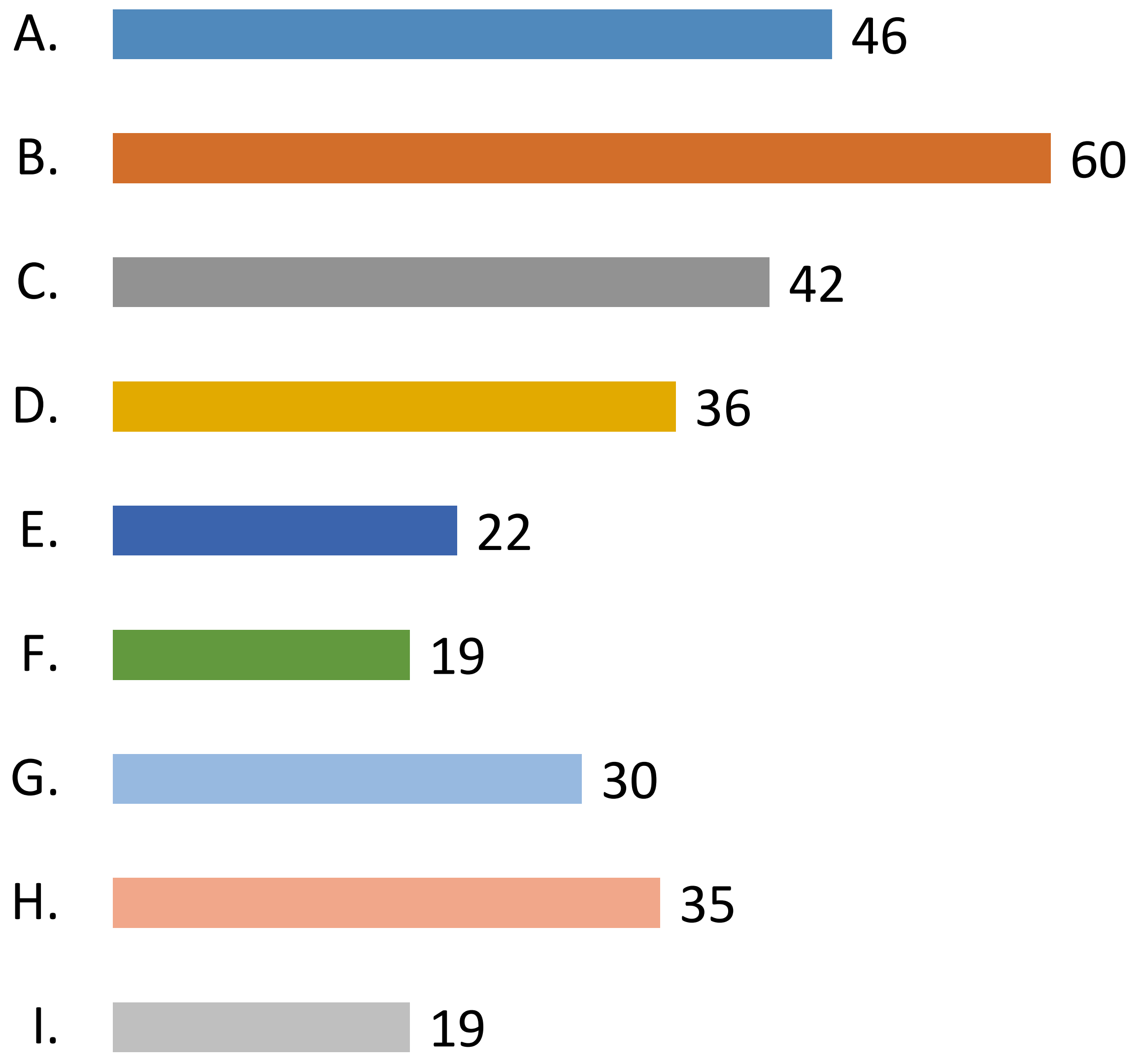
- Will be covered in today's lecture
- Make sure you'll know their meaning!

- Translational motion
- Rotational motion
- Arc length, $s = r\theta$
- Angular position (angle), θ
- Angular velocity, ω
- Angular acceleration, α
- Torque, τ
- Moment of inertia, I
- Newton's 2nd Law for rotation, $\tau = I\alpha$

Textbook Chapter 10

Which concepts are you already familiar with?

A. Translational motion
B. Rotational motion
C. Arc length, $s = r\theta$
D. Angular position (angle), θ
E. Angular velocity, ω
F. Angular acceleration, α
G. Torque, τ
H. Moment of inertia, I
I. Newton's 2^{nd} Law for rotation, $\tau = I\alpha$



Rotational kinematics

Translational kinematics

For movement from one point to another, we should use the **kinematic equations**:

$$x = x_i + v_i t + \frac{1}{2}at^2$$

$$v = v_i + at$$

$$v^2 = v_i^2 + 2a(x - x_i)$$

Moving from one point to another is often called "translation" so these are known as the translational kinematic equations.

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Moving from one point to another is often called "translation" so these are known as the translational kinematic equations.

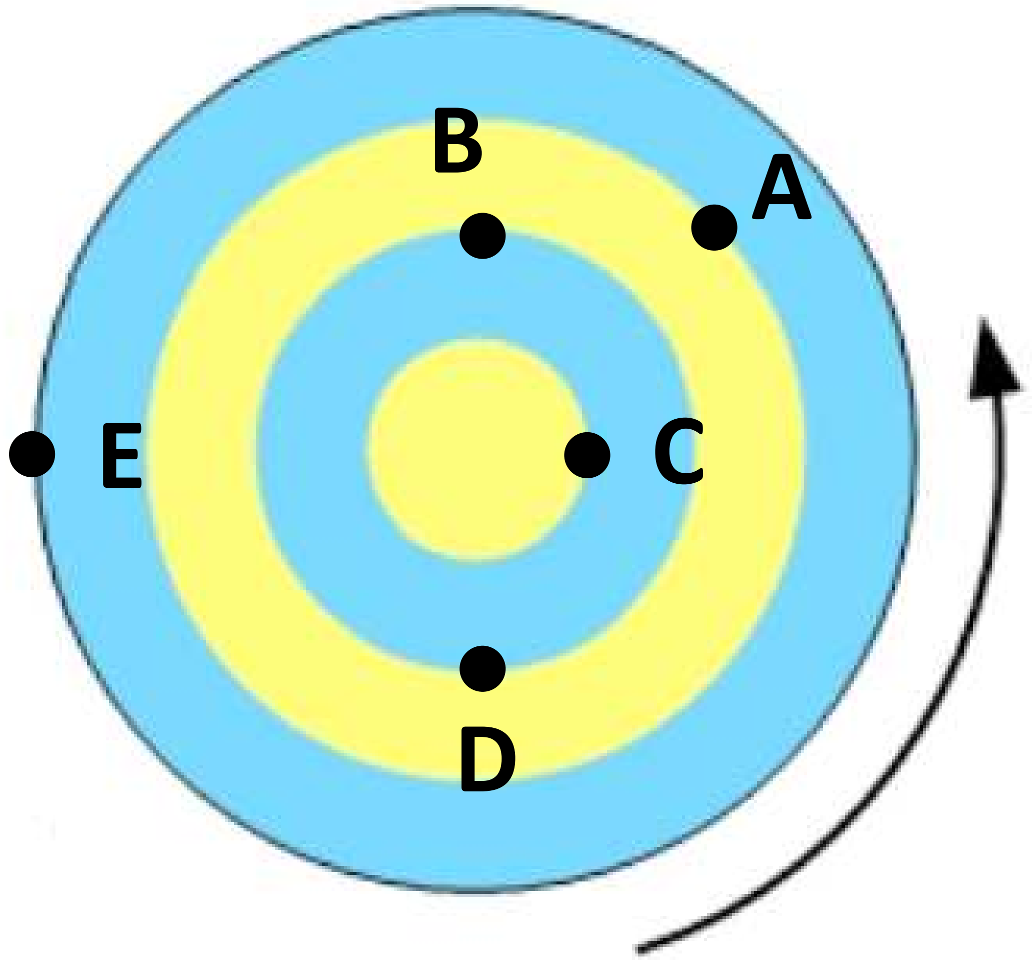
In this lecture we shall consider a special type of movement:

rotation

(movement about some fixed point)

Rotating disk

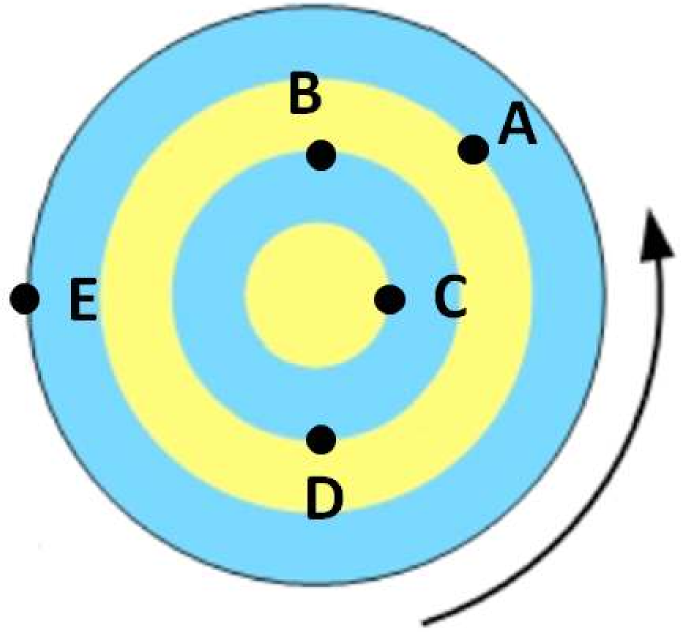
Consider five points on a moving disk.

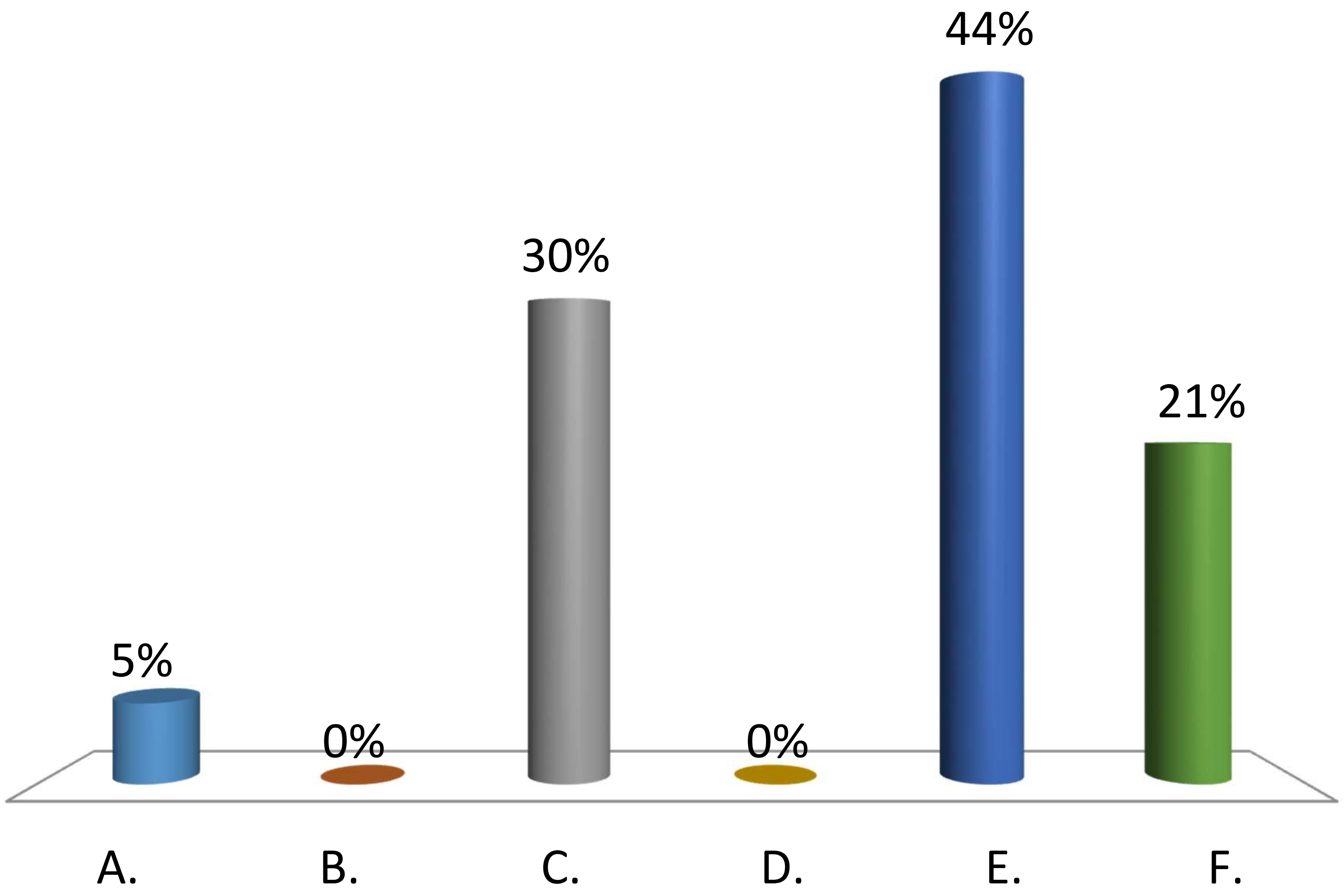


Which of these points moves the fastest?
(which has the largest speed v?)

Which point moves the fastest?

A. Point A has the largest speed
B. Point B has the largest speed
C. Point C has the largest speed
D. Point D has the largest speed
E. Point E has the largest speed
F. All move with the same speed





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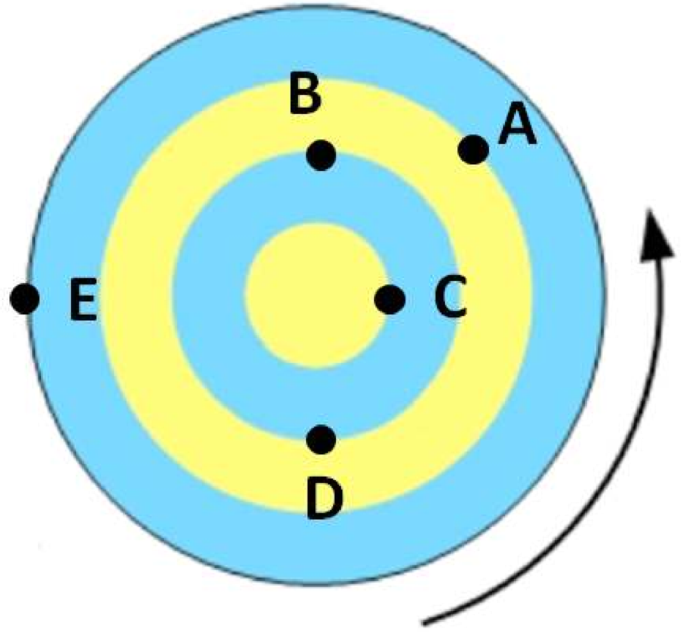
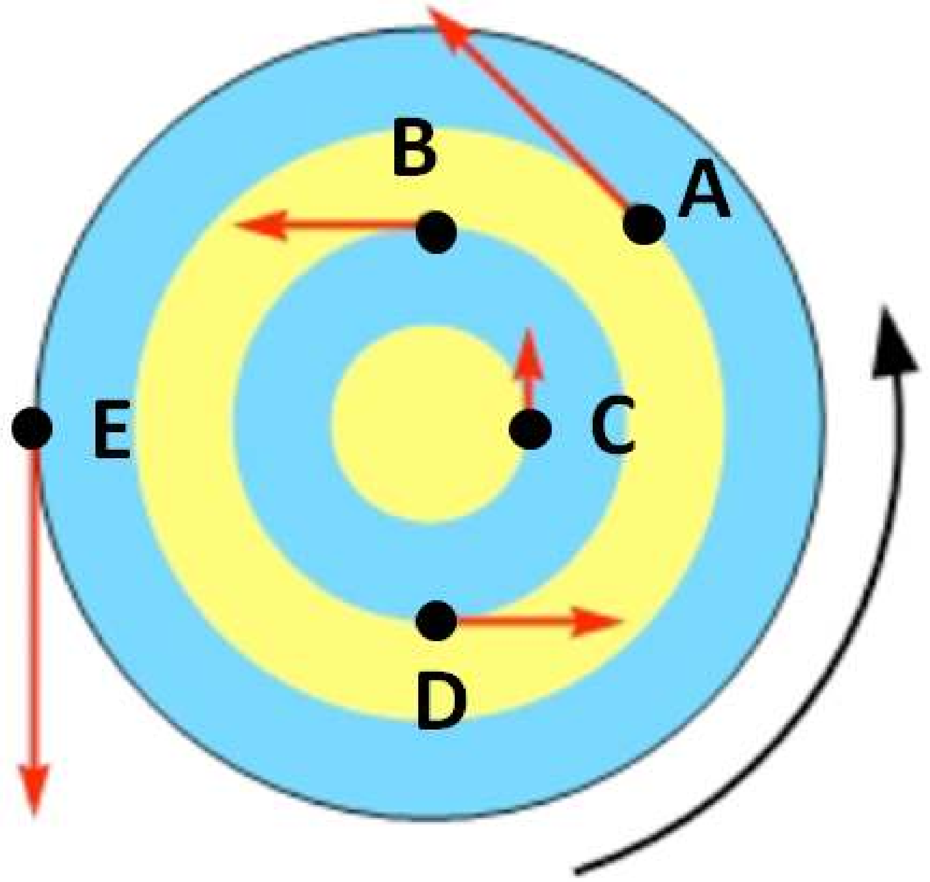


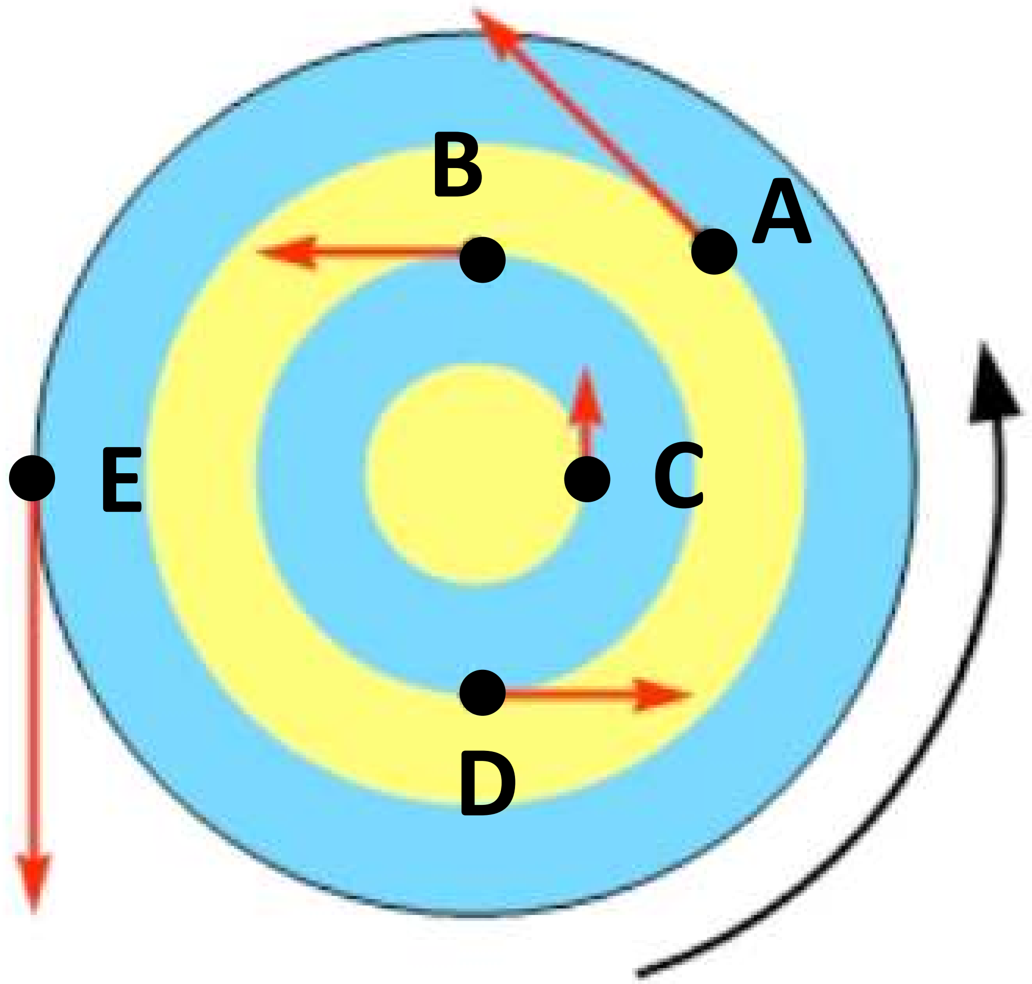


Speed of each point depends on how far from the center you are.

At the center, you rotate but your speed v is zero!

Rotating disk

How fast is this disk moving?



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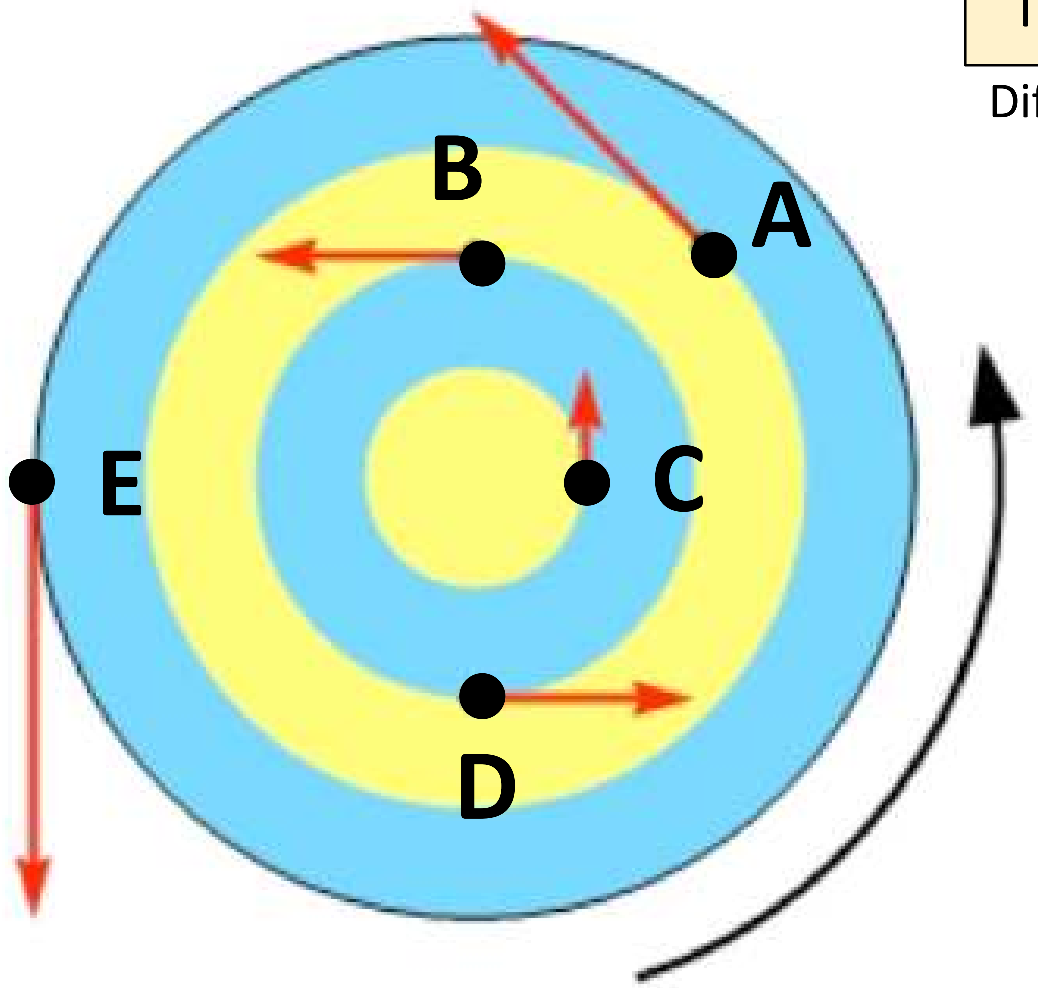

Talking about the speed (m/s) of the disk makes no sense here

Different points on disk have different speeds...

Rotating disk

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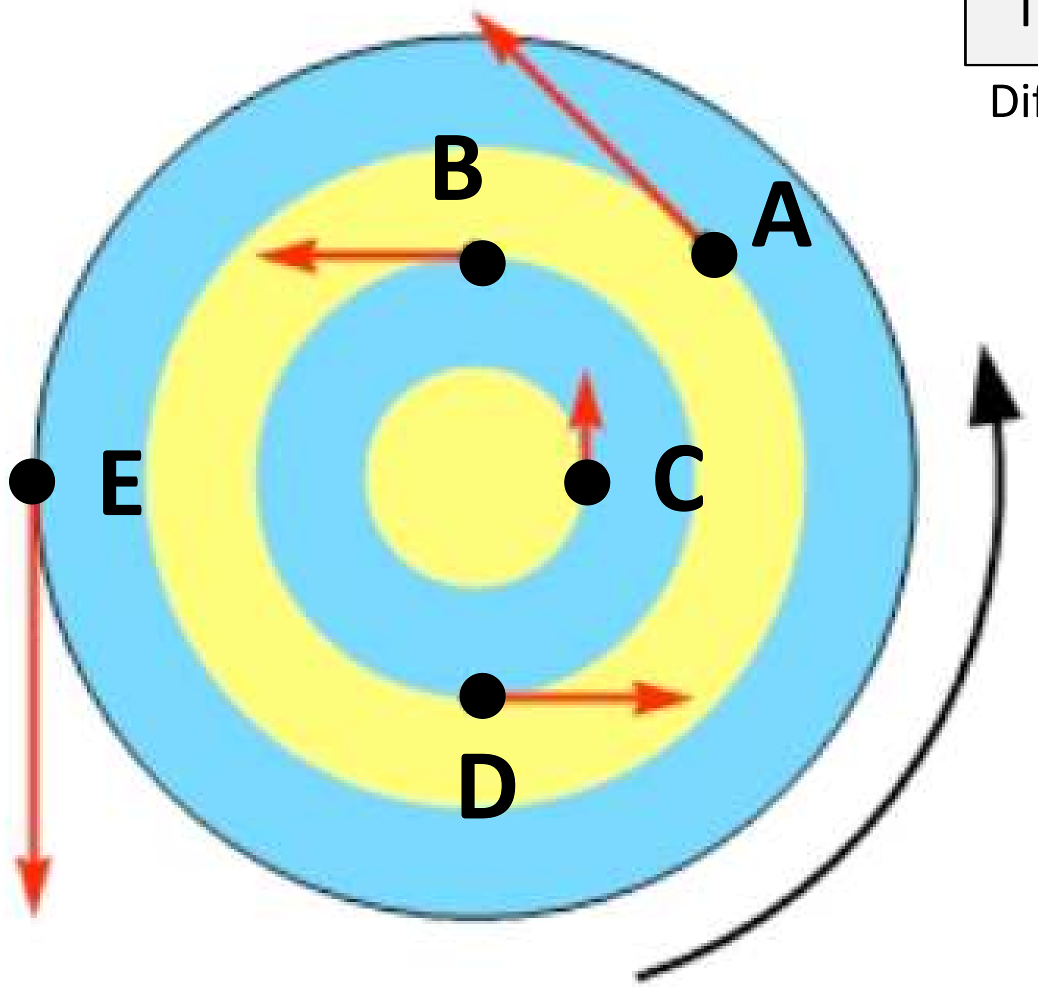

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Different points on disk have different speeds...

You need to tell me, for example:

- "number of turns per minute", or
- "number of revolutions per second", or
- "number of degrees it turns per second", or
- "number of radians it turns per second"

1 revolution = 360° = 2π rad

Rotating disk

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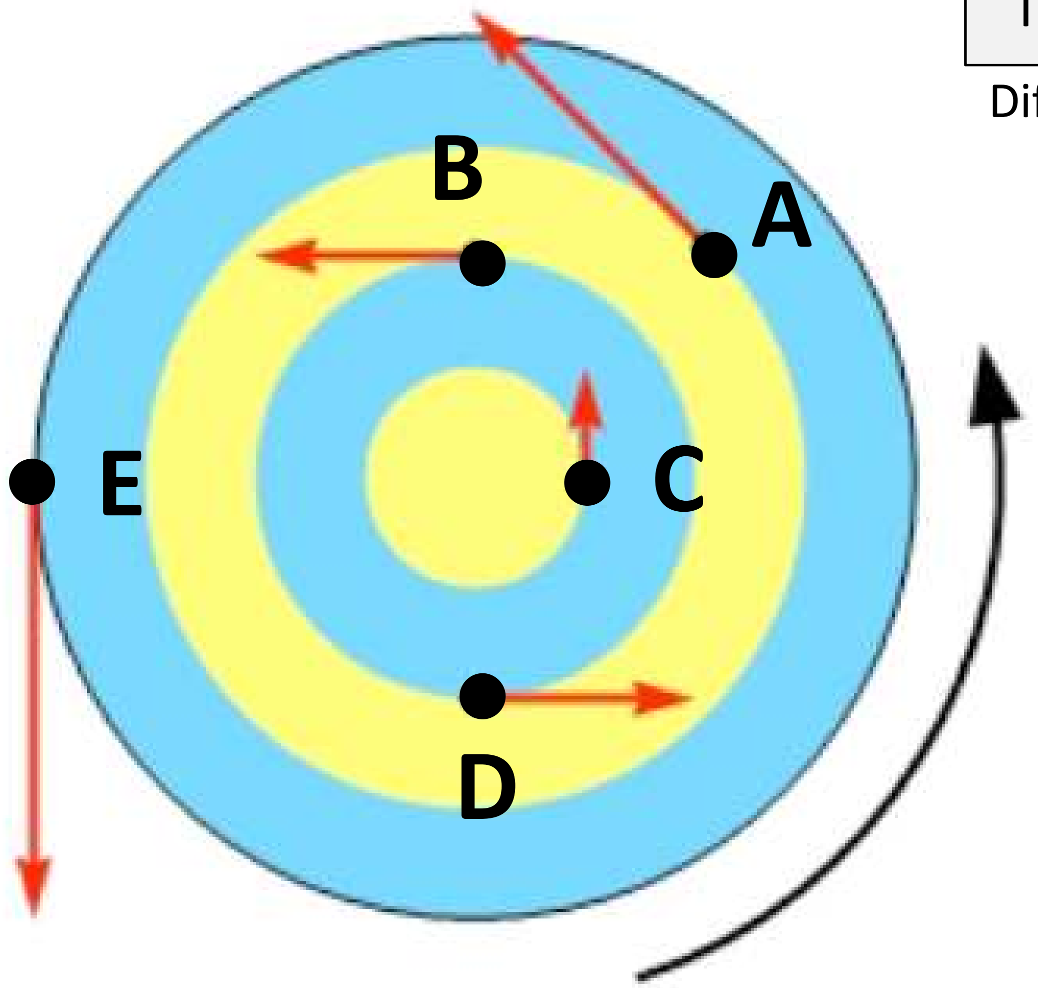

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The standard unit is radians/second (rad/s)

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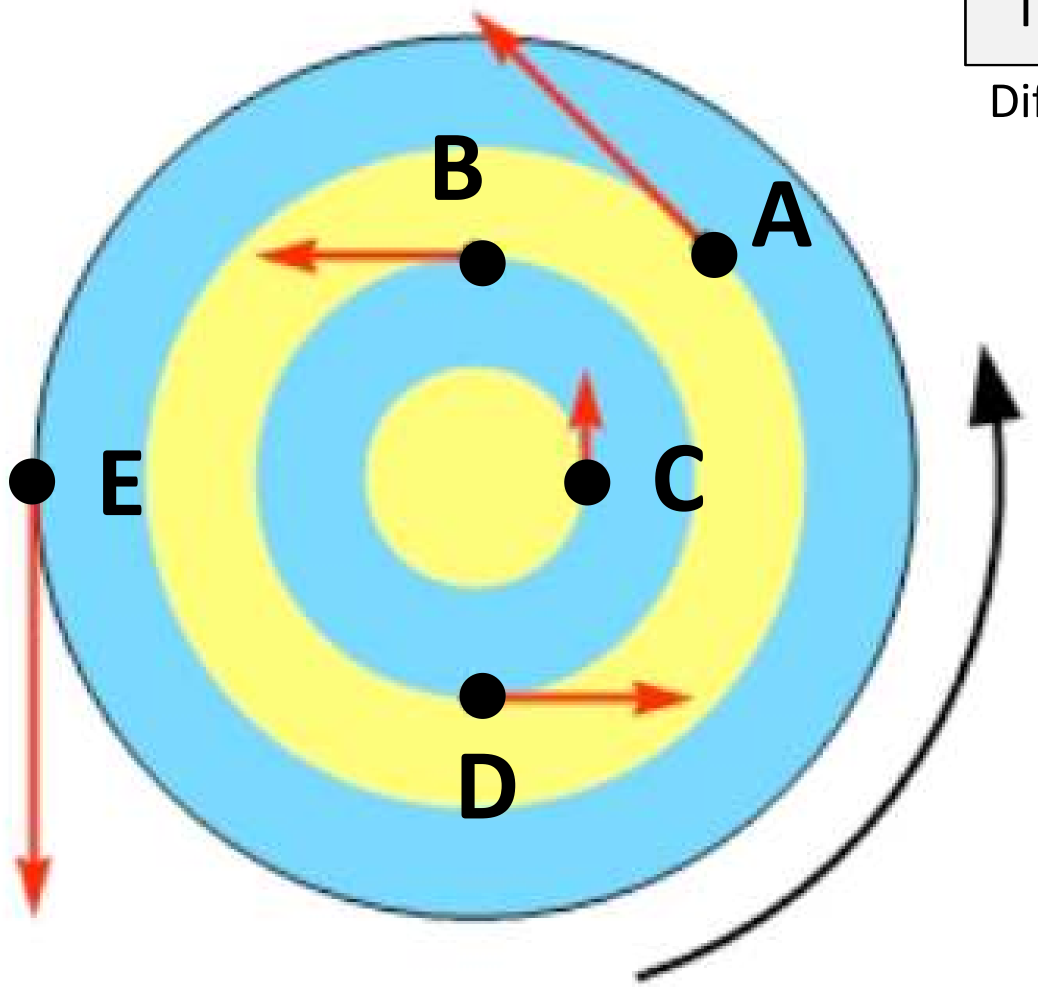

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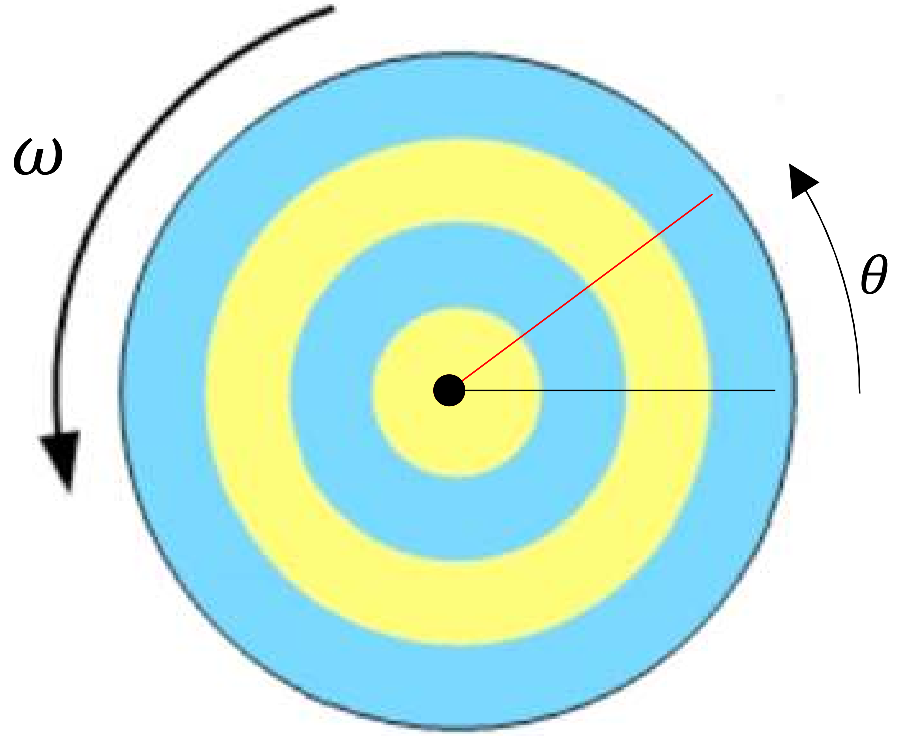
Angular speed ω and angular position θ

Angular speed ω is how much the angle θ changes with time:

$$\omega \approx \frac{\Delta\theta}{\Delta t}$$

The exact definition is: $\omega = \dfrac{d\theta}{dt}$ = slope in a θ-vs-t graph



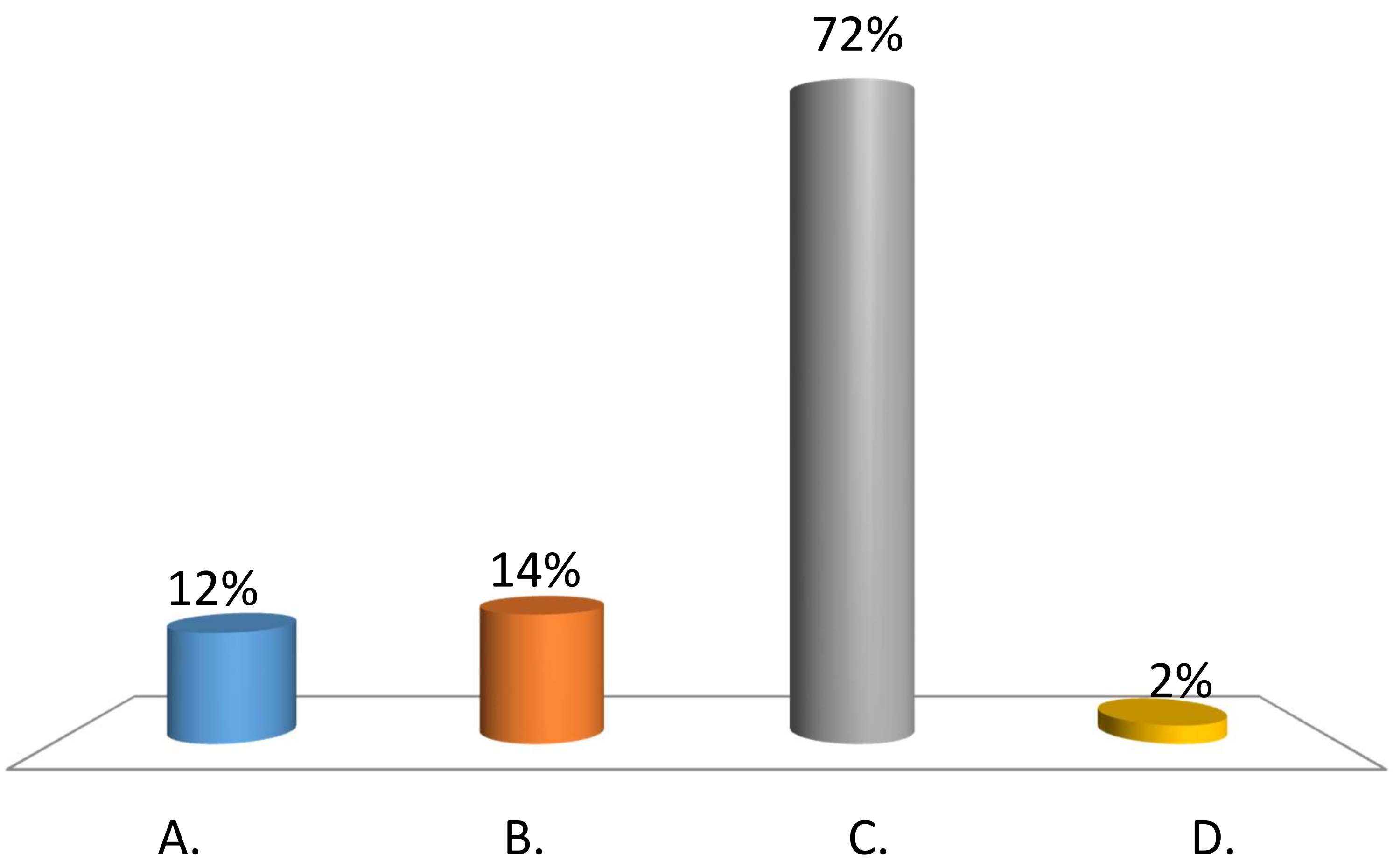
The standard unit for angular position θ is radians (rad)

The standard unit for angular speed ω is radians/second (rad/s)

How much has the disk rotated after 20 sec?

A disk rotates with a constant angular speed:
ω = 6 revolutions per minute (1 min = 60 sec)
How much has the disk rotated after 20 seconds?

A. Exactly 1/2 revolution (180° or 3.14 rad)
B. Exactly 1 revolution (360° or 2π rad)
C. Exactly 2 revolutions (720° or 4π rad)
D. Exactly 4 revolutions (1440° or 8π rad)



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20 seconds is exactly 1/3 of one minute

So the disk rotates 2 full revolutions:

$\theta = 4\pi$ radians

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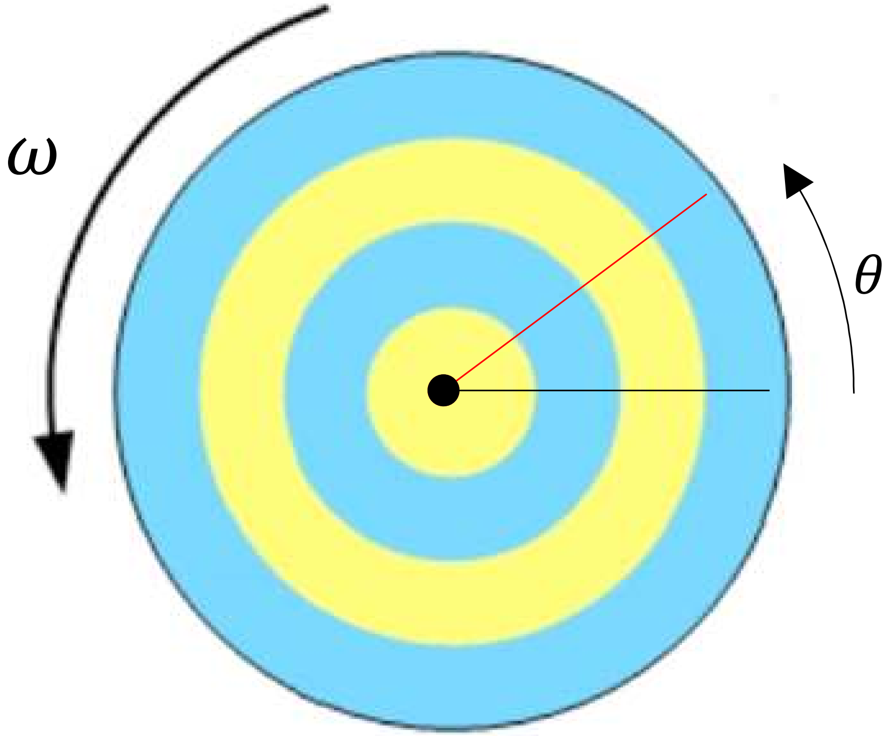

The angular position θ at time t is: $\theta = \theta_i + \omega t$

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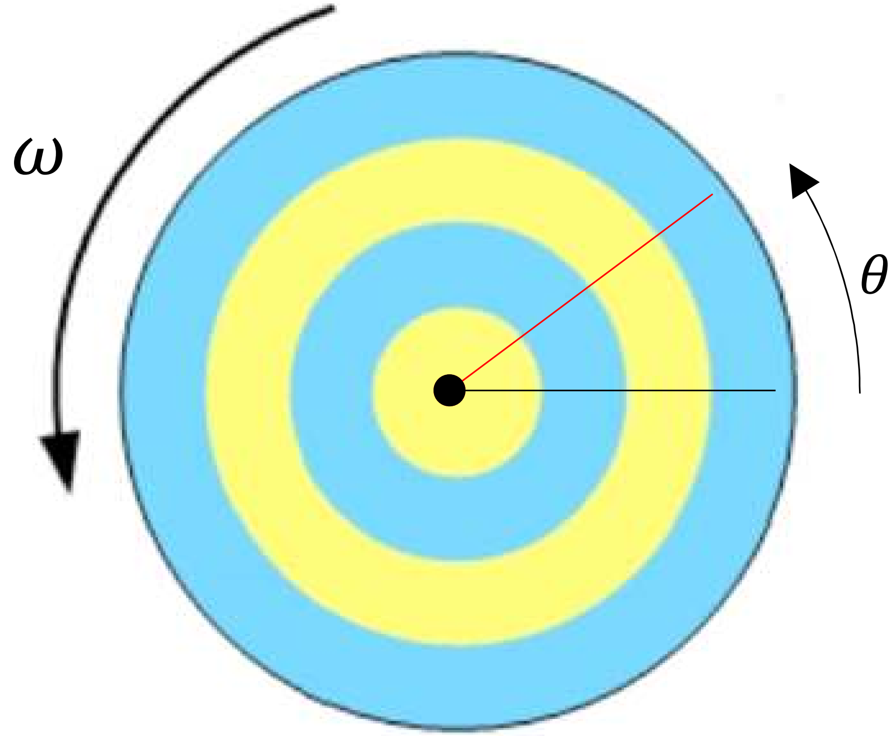

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So the disk rotates 2 full revolutions:

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The angular position θ at time t is: $\theta = \theta_i + \omega t$

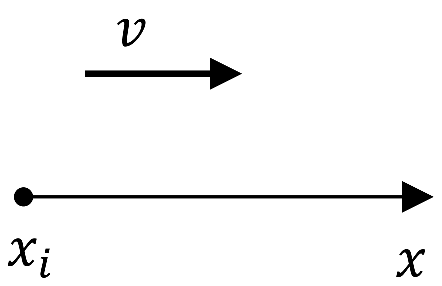
ω = 6 rev/min = $(6 \times 2\pi$ rad)/(60 seconds) = 0.628 rad/s

At $t = 20$ sec: $\theta = 0 + (0.628)(20) = 12.59$ rad = 4π rad

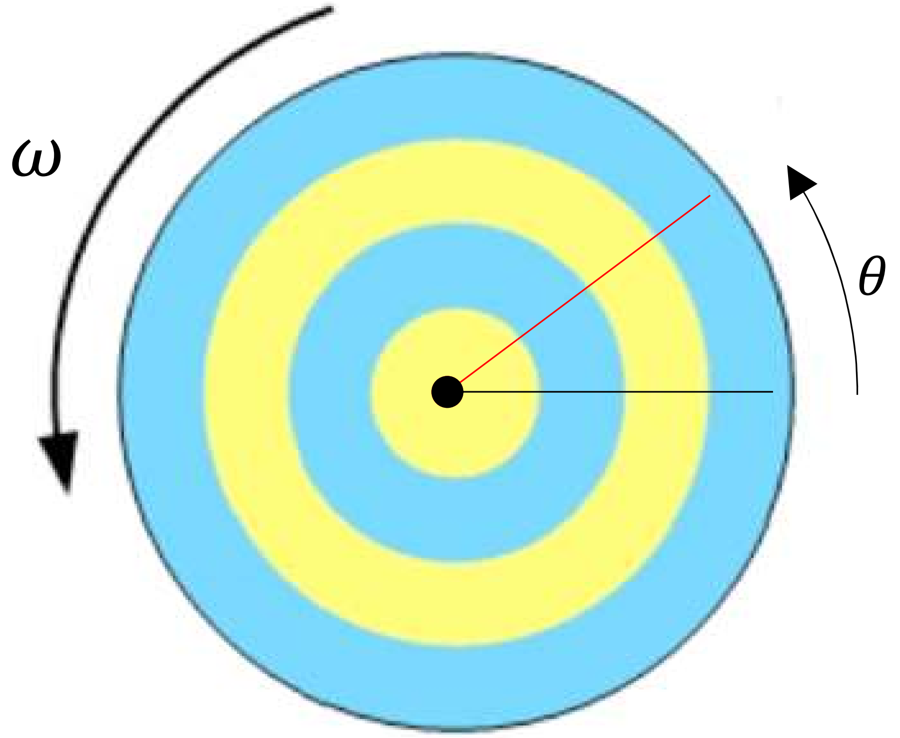
Similarity between translational and rotational

- Notice how similar these equations are:

Moving from position x_i to position x:



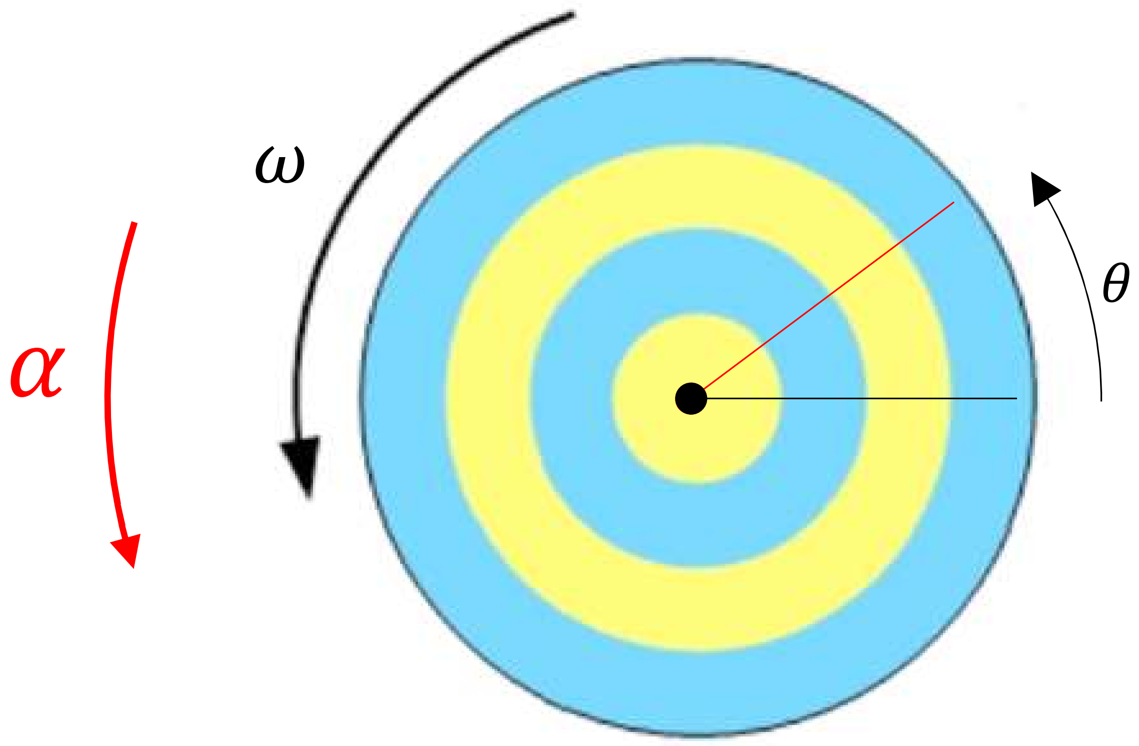
$$x = x_i + vt$$



The angular position θ at time t is: $\theta = \theta_i + \omega t$

Angular acceleration α

- A disk can rotate faster and faster – it can have rotational acceleration!



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"alpha" (lowercase Greek letter)

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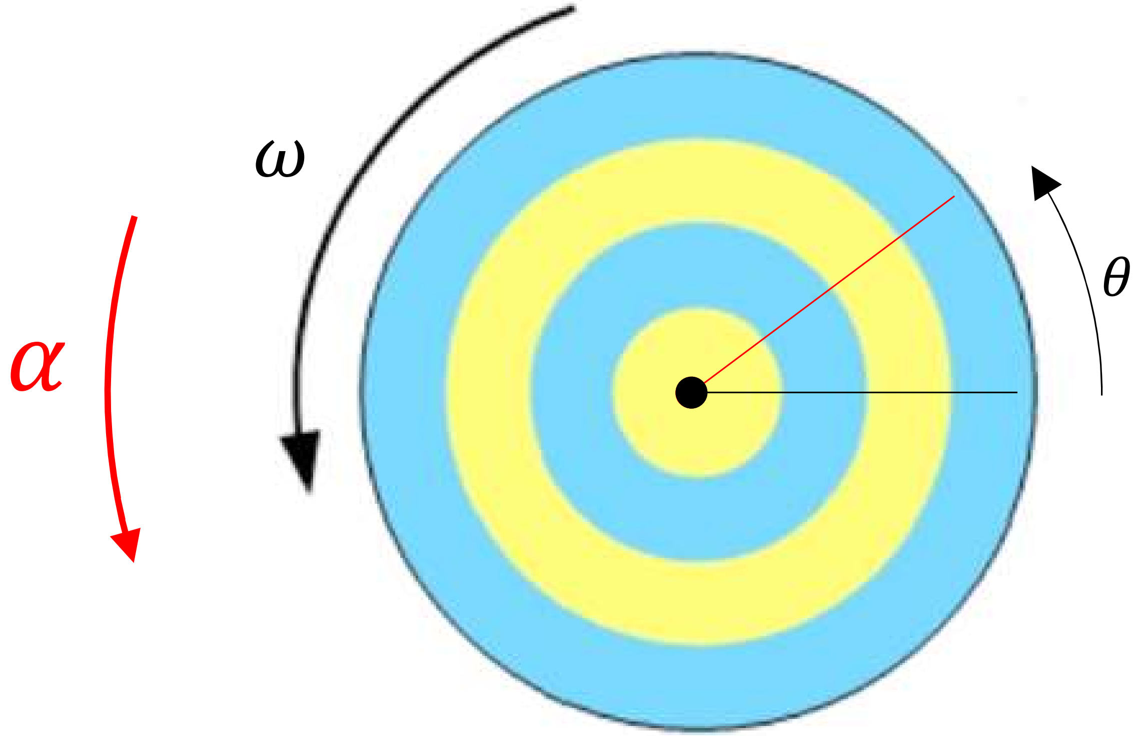

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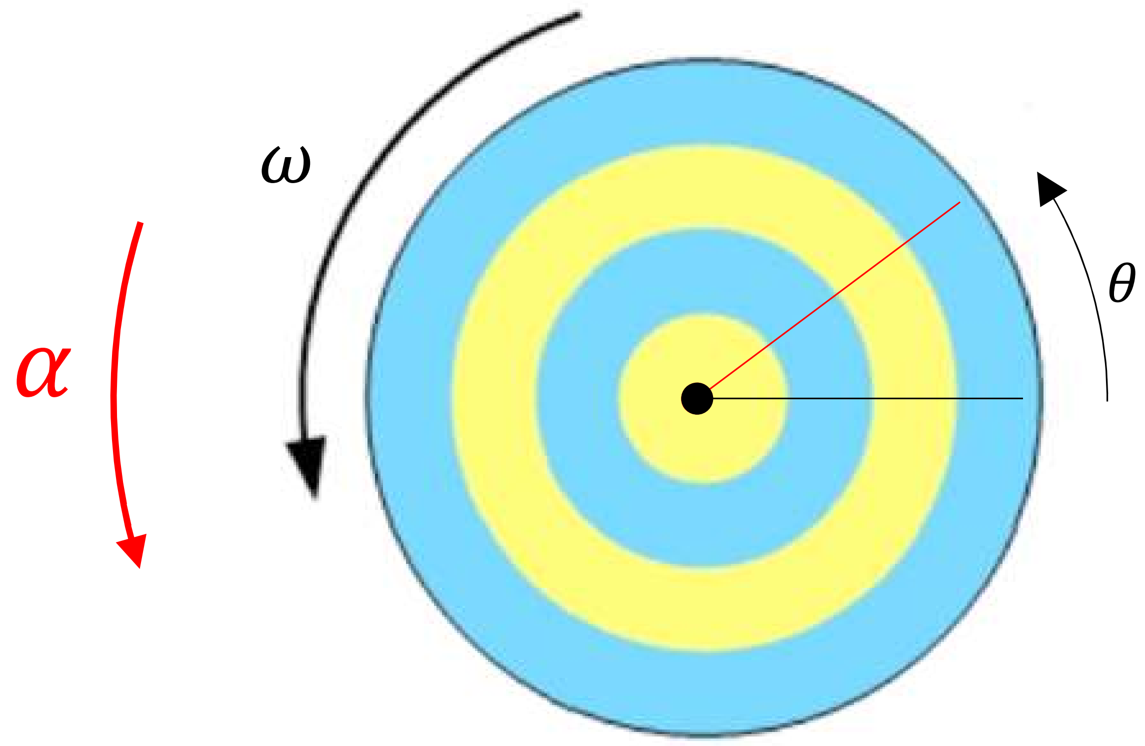
- This means that the angular speed ω increases:

$$\omega = \omega_i + \alpha t$$

(assuming constant angular acceleration α)



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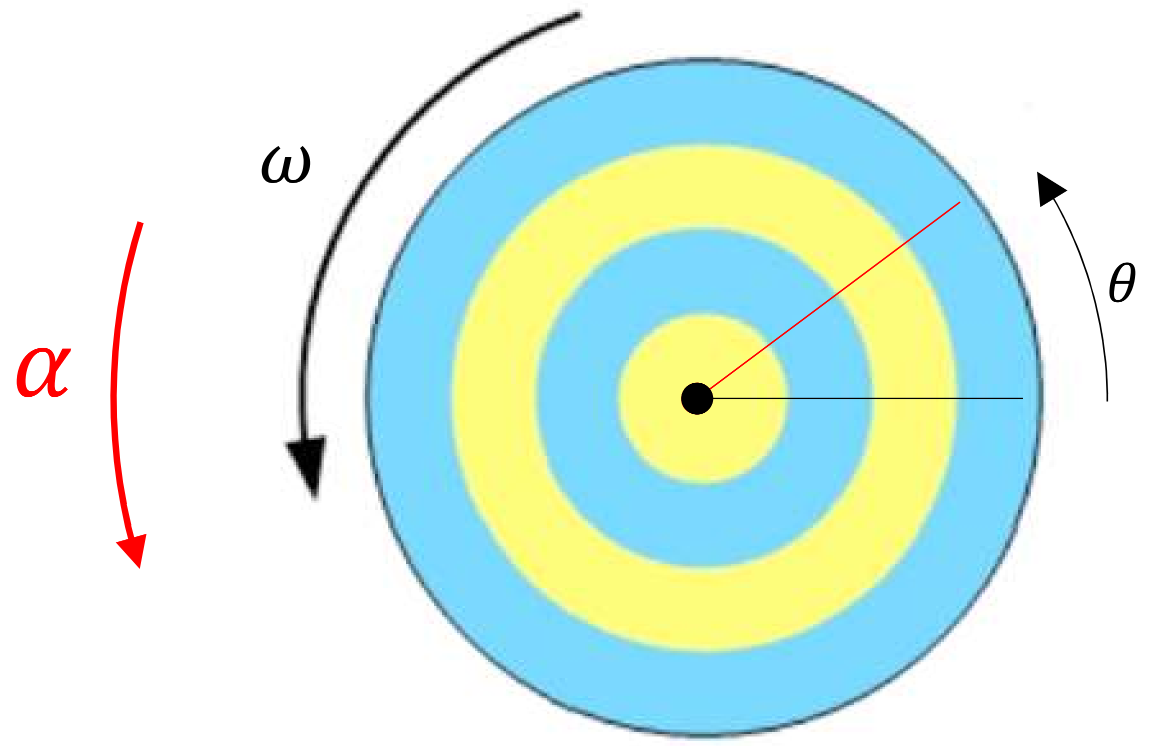


- Units of angular acceleration are rad/s^2

Rotational kinematic equations

For "translational" movement, you need to use the **translational kinematic equations**:

$$x = x_i + v_i t + \tfrac{1}{2} a t^2$$

$$v = v_i + at$$

$$v^2 = v_i^2 + 2a(x - x_i)$$

x in m (meters)
v in m/s
a in m/s^2

For rotational movement, you need to use the **rotational kinematic equations**:

$$\theta = \theta_i + \omega_i t + \tfrac{1}{2} \alpha t^2$$

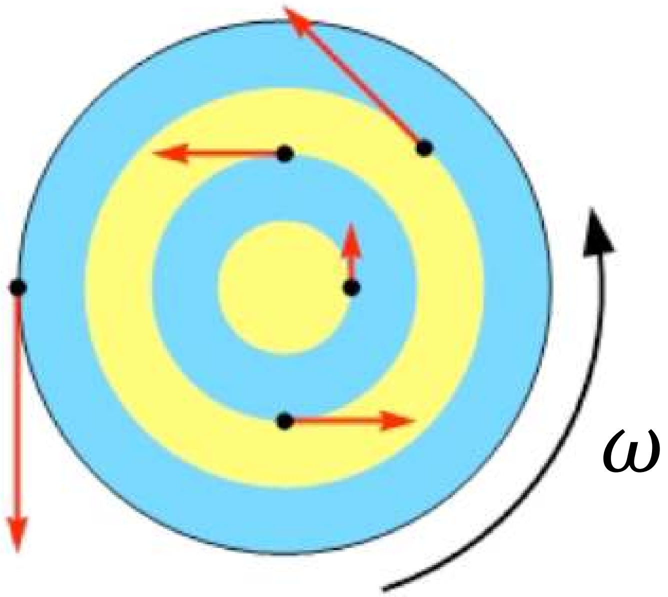
$$\omega = \omega_i + \alpha t$$

$$\omega^2 = \omega_i^2 + 2\alpha(\theta - \theta_i)$$

θ in rad (radians)
ω in rad/s
α in rad/s^2

Sign convention for θ, ω and α

Important: we typically define counter-clockwise to be positive.



angular velocity is
positive in this case
$(\omega > 0)$

Yes, that's a little counter-intuitive.

Example problem!

- A dentist drill is rotating with angular speed of $\omega = 4.0$ rad/s

- Because of friction the drill slows down with a constant angular acceleration α
  - Note: α must be negative if we take $\omega > 0$

- After 12 seconds the drill stops rotating.

- How large must α have been?

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Because of friction the drill slows down with a constant angular acceleration α
After 12 seconds the drill stops rotating.
How large must α have been?

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Which kinematic equation should we use?

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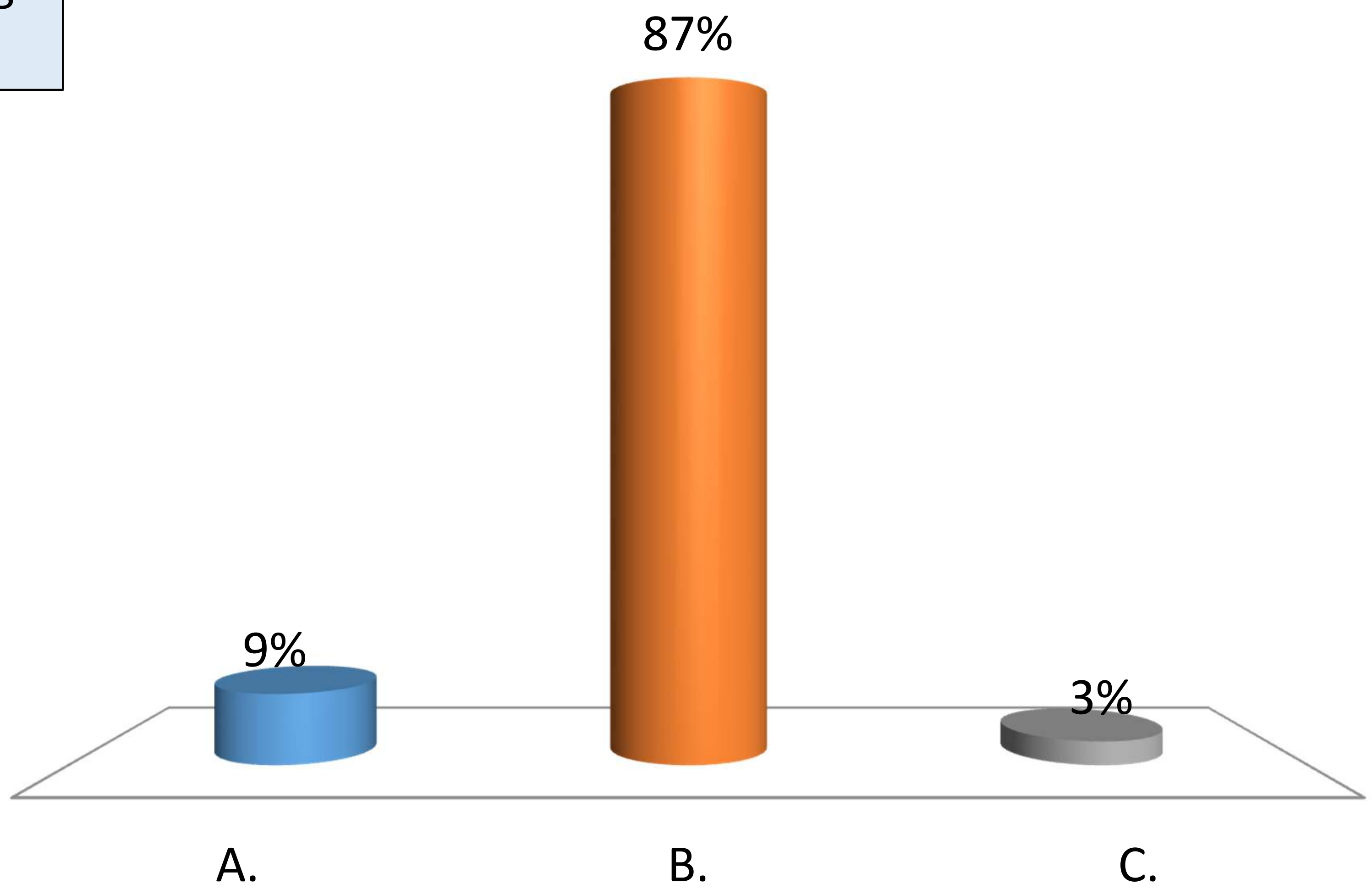

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$$(0) = (4.0) + \alpha(12)$$

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$$(0) = (4.0) + \alpha(12)$$

$$\alpha = -4.0/12 = -0.33 \text{ rad/s}^2$$

Radial and tangential motion

- Coordinate systems are important!
- With rotating systems it is hard to talk about x and y ...
- Instead we use the terms radial and tangential
- For example: radial velocity v_r and tangential velocity v_t
  - Imagine walking on a rotating disk toward the center with speed v_r

Radial and tangential motion

Tangential speed of point on disk:

$$v_t = r\omega$$

with r distance from center

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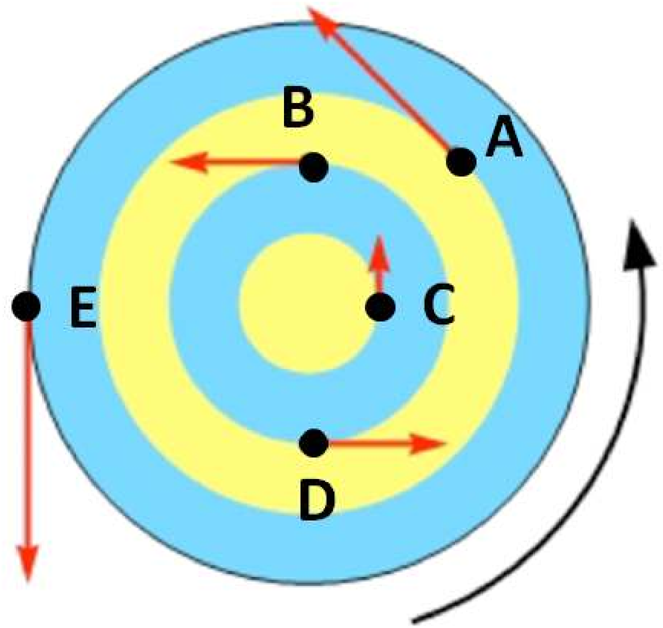

All points here have a tangential velocity v_t but no radial velocity v_r.

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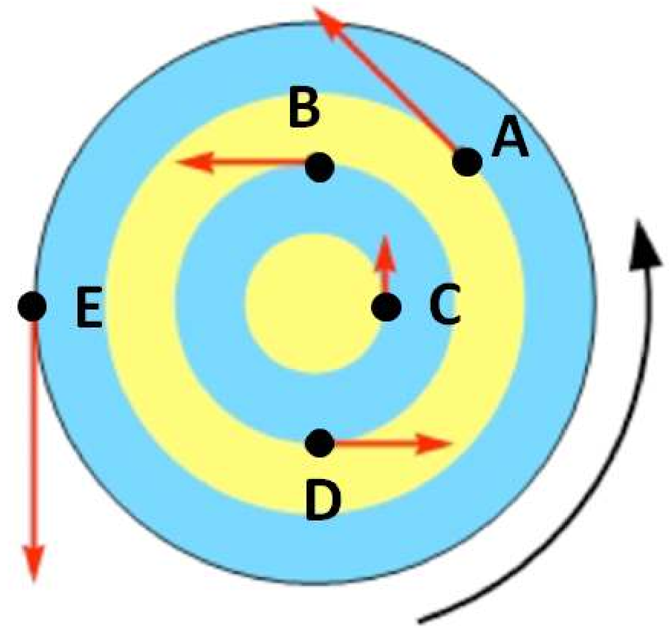
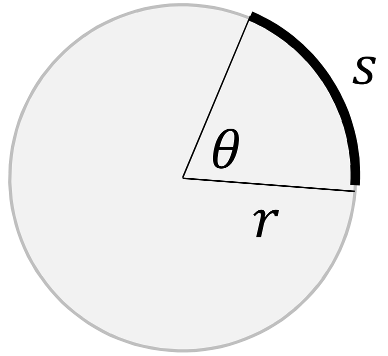


All points here have a tangential velocity v_t but no radial velocity v_r.

Good news: we typically assume zero radial speed (makes problems a lot easier)

Tangential motion

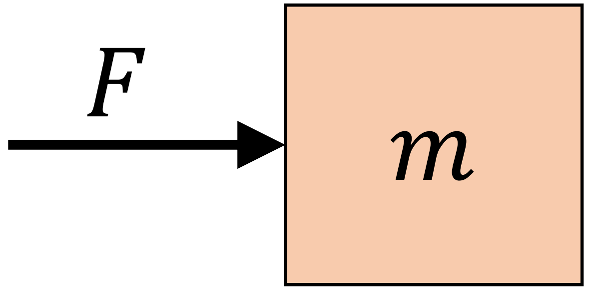


- Arc length = $s = r\theta$ (an arc is part of a circle)
- Tangential velocity = $v_t = r\omega$
- Tangential acceleration = $a_t = r\alpha$
- Radial (or centripetal) acceleration = $a_c = v^2/r$

Torque

How can we make an object rotate?

To make an object move, we need to apply a force F



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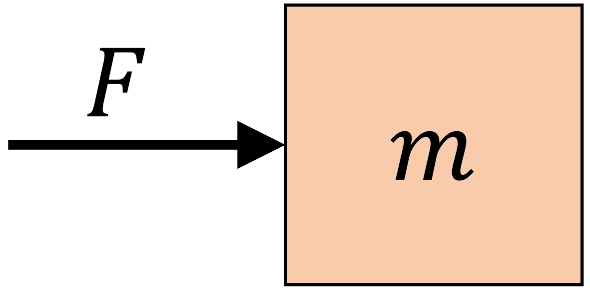
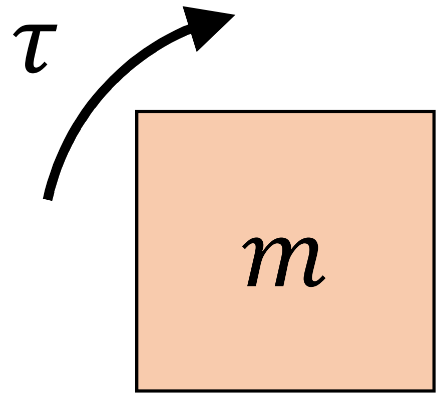


To make an object rotate, we need to apply a torque τ



- But... More about that on Friday!

Thank you!

See you next time: Friday 24 March at 2:30pm