Lecture 27:

Rotational Kinetic Energy and Angular Momentum

Making you see the world from a different angle:

Erik Lascaris

Announcements

- Homework #9 (on paper) was due yesterday!
  - Will be graded within the next couple of days
  - Practice version of homework will be made available on WebAssign.

- Homework #10: due Tue Apr 4th (1 day before midterm!)
  - Warning: No extension possible for HW 10 !

- Lab #6 (Torque & Moment of Inertia) this week
  - Hope you did your pre-lab!

General structure of the midterm

- 7 problems in total

- Problem 1, worth 10 points:
  - Multiple Choice questions about concepts and some stuff you should memorize

- Problems 2, 3, 4, 5, 6, 7, each worth 15 points:
  - Similar problems as on quizzes and homework
    - Conservation of energy
    - Conservation of momentum
    - Rotational kinematics
    - Static equilibrium (sum of forces is zero & sum of torques is zero)
    - Torque and rotational motion (using $\tau_{\text{net}} = I\alpha$)
    - ??? (conservation of angular momentum? Universal gravity? More of the other stuff?)

- Sums up to 100 points in total

How to prepare for the midterm

Midterm 2: WEDNESDAY (Apr 5th) at 6:30pm in PHO-206

- Try to do the quizzes again, and understand the solutions *solutions = soon on Piazza*
  - Quiz 4: introducing energy
  - Quiz 5: conservation of energy
  - Quiz 6: conservation of momentum
  - Quiz 7: rotational kinematics
  - Quiz 8: torque and/or moment of inertia and/or conservation angular momentum (?)
- Practice the homework problems – worked-out solutions are on Piazza! *solutions = soon on Piazza*
  - HW-6, problems 2, 5, 7, 9
  - HW-7, problems 2, 4, 5
  - HW-8, problem 1
  - HW-9, problems 5, 7, 8
  - HW-10, problems 2, 4, 5, 6 = available at last minute: after Tuesday midnight!
- Go to Piazza, and make sure you understand slides
- Do the practice exams
  - Midterm 2 of Fall 2012 and Fall 2013 and Fall 2014 are on Piazza!
- Check out the list on Piazza: Things_you_should_know_for_Test2.pdf *NEW on Piazza*
- Memorize the "List of things to memorize"

Things you need to memorize!

These things are literally Problem 1 on midterm!

- And check out the list of things in Things_you_should_know_for_Test2.pdf --- soon on Piazza!

Today's Concepts

- Will be covered in today's lecture
- Make sure you'll know their meaning!

- Linear kinetic energy ($K = \frac{1}{2}mv^2$)
- Rotational kinetic energy ($K_{\mathrm{rot}} = \frac{1}{2}I\omega^2$)
- Angular velocity, ω
- Moment of inertia, I
- Angular momentum, L
- Conservation of angular momentum, $L_i = L_f$

Textbook Chapter 11

Which concepts are you already familiar with?

A. Linear kinetic energy ($K = \frac{1}{2}mv^2$)

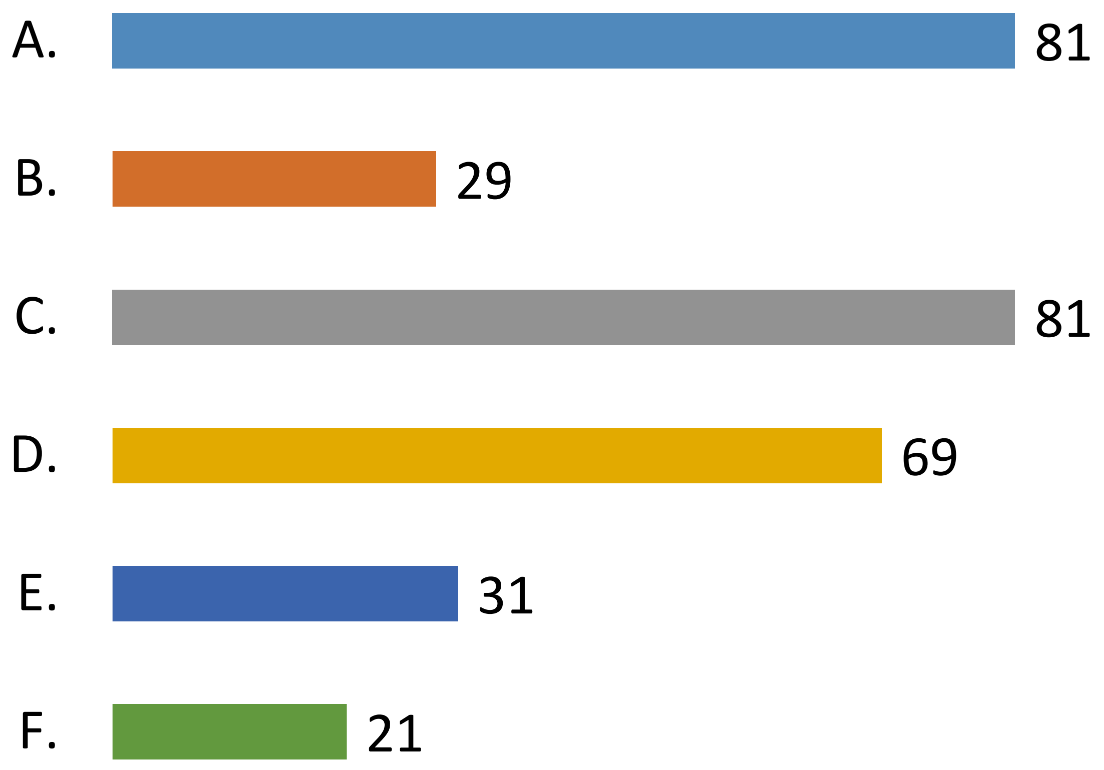
B. Rotational kinetic energy ($K_{\text{rot}} = \frac{1}{2}I\omega^2$)

C. Angular velocity, ω

D. Moment of inertia, I

E. Angular momentum, L

F. Conservation of angular momentum, $L_i = L_f$



Quick reminder:
radians, torque, moment of inertia

The special unit "radians"

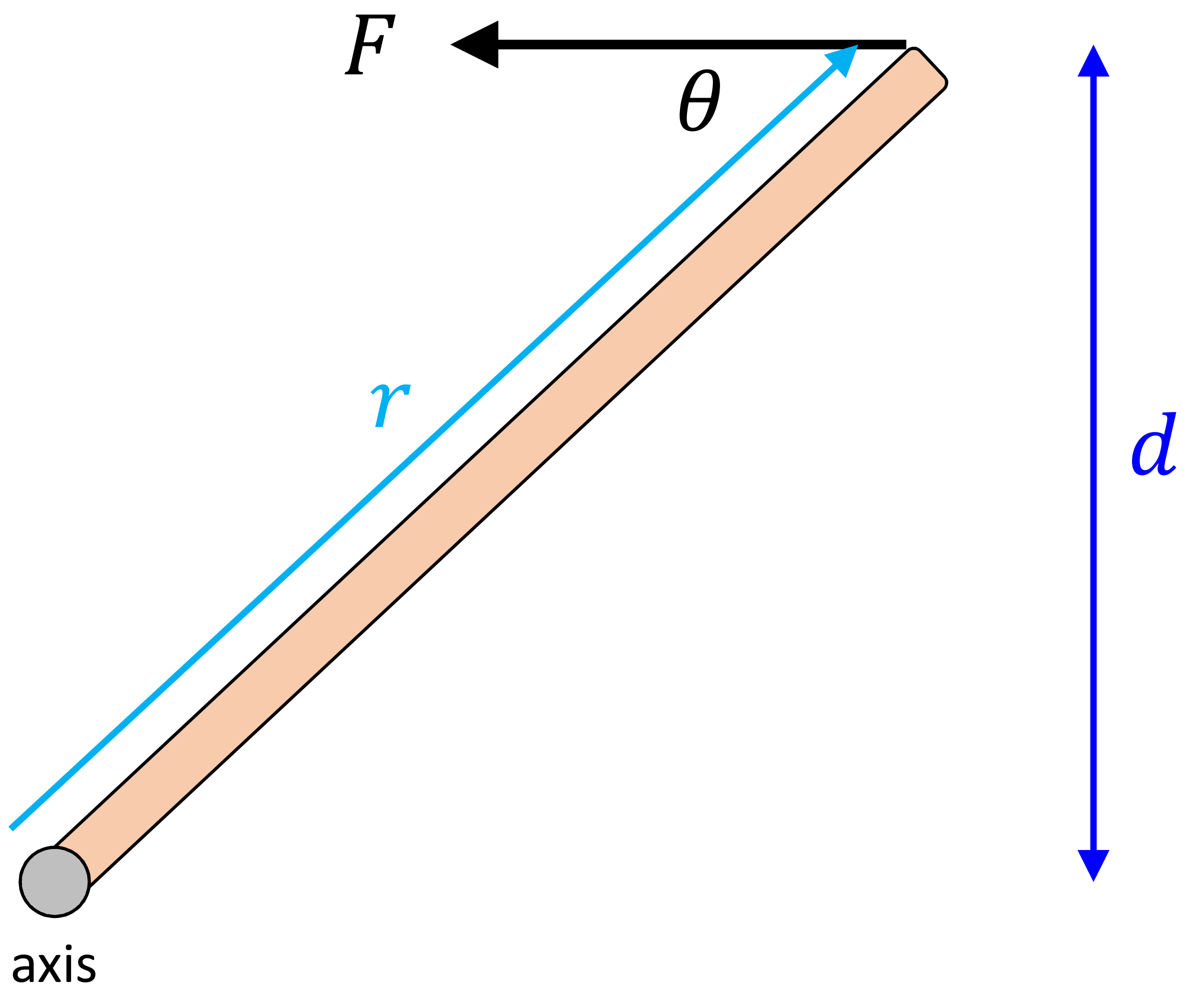
- Several people hit on this problem:

  what is the unit for angular acceleration α (alpha) if $\tau = I\alpha$???

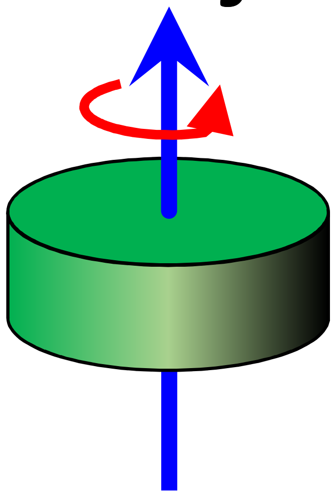
  - The standard unit for moment of inertia I is: kg m^2
  - The standard unit for torque τ (tau) is: N m
- RULE: If you use the correct equation and plug in values with the right units, then what comes out will automatically have the right units as well.
- Using $\tau = I\alpha$ you'd get: [N m] = [kg m^2] [alpha] is same as [kg m/s^2] = [kg m^2] [alpha] so the units of alpha would be $1/s^2$??
- Actually the standard unit for angular acceleration α (alpha) is: rad/s^2
- Reason: "radians" (rad) is not a real unit. For example in arc length $s = r\theta$ both s and r are in meters, while θ is in radians!

Torque τ

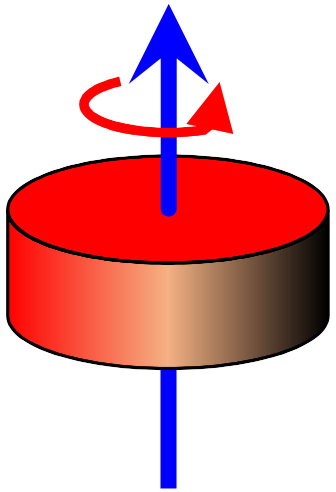
- Often the easiest way to get torque is by finding the arm length d.
  $\tau = Fd$
- If instead you know r and θ it makes sense to uses
  $\tau = Fr\sin\theta$



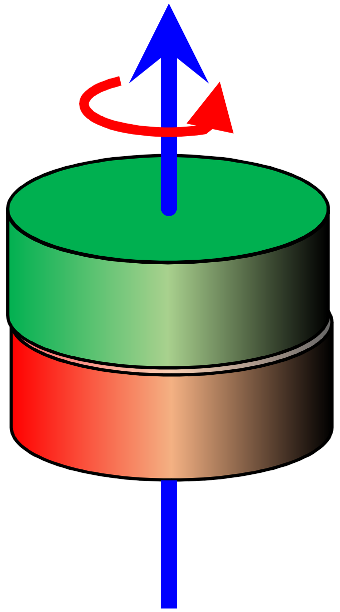
Moment of inertia of two objects together



- If disk 1 has moment of inertia I_1



- And disk 2 has moment of inertia I_2
- Then the moment of inertia of both disks glued together is: $I_{12} = I_1 + I_2$



Rotational Kinetic Energy

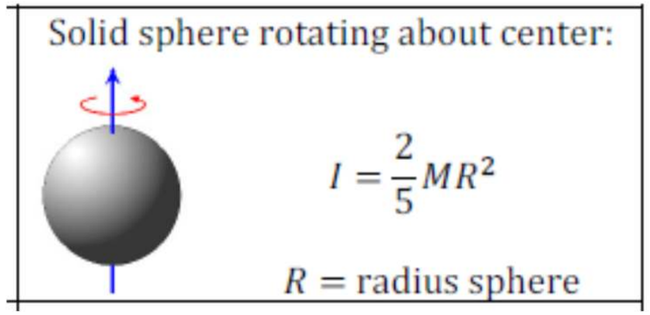
Rotational kinetic energy

- In addition to regular kinetic energy $K = \frac{1}{2}mv^2$,
  an object can also have rotational kinetic energy $K_{rot} = \frac{1}{2}I\omega^2$

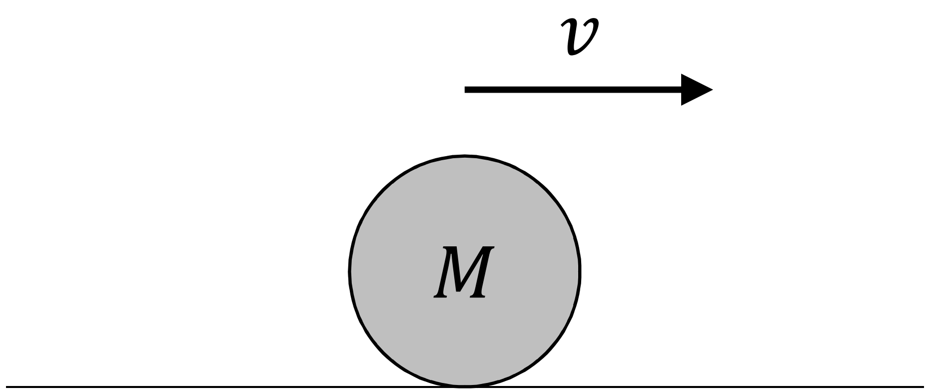
- A disk spinning at ω rad/s carries a kinetic energy of $K = \frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega^2$
  - You could also chop up the disk in tiny pieces, and consider the speed of each little piece – the total kinetic energy would be the same.

- An object can also have both linear and rotational kinetic energy
  - For example: a ball that is rolling.

Rolling ball

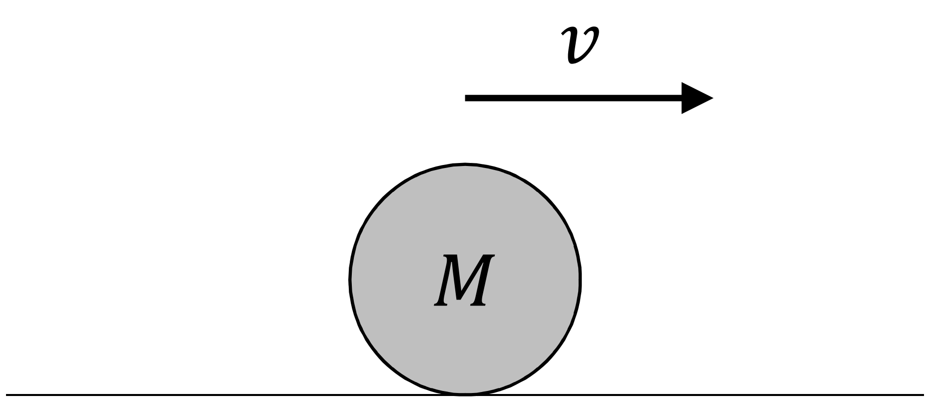


- Imagine a ball rolling on the floor.
- The ball moves with a speed v.
- If the ball is rolling without slipping then it is also rotating!
- With what angular speed does the ball rotate?

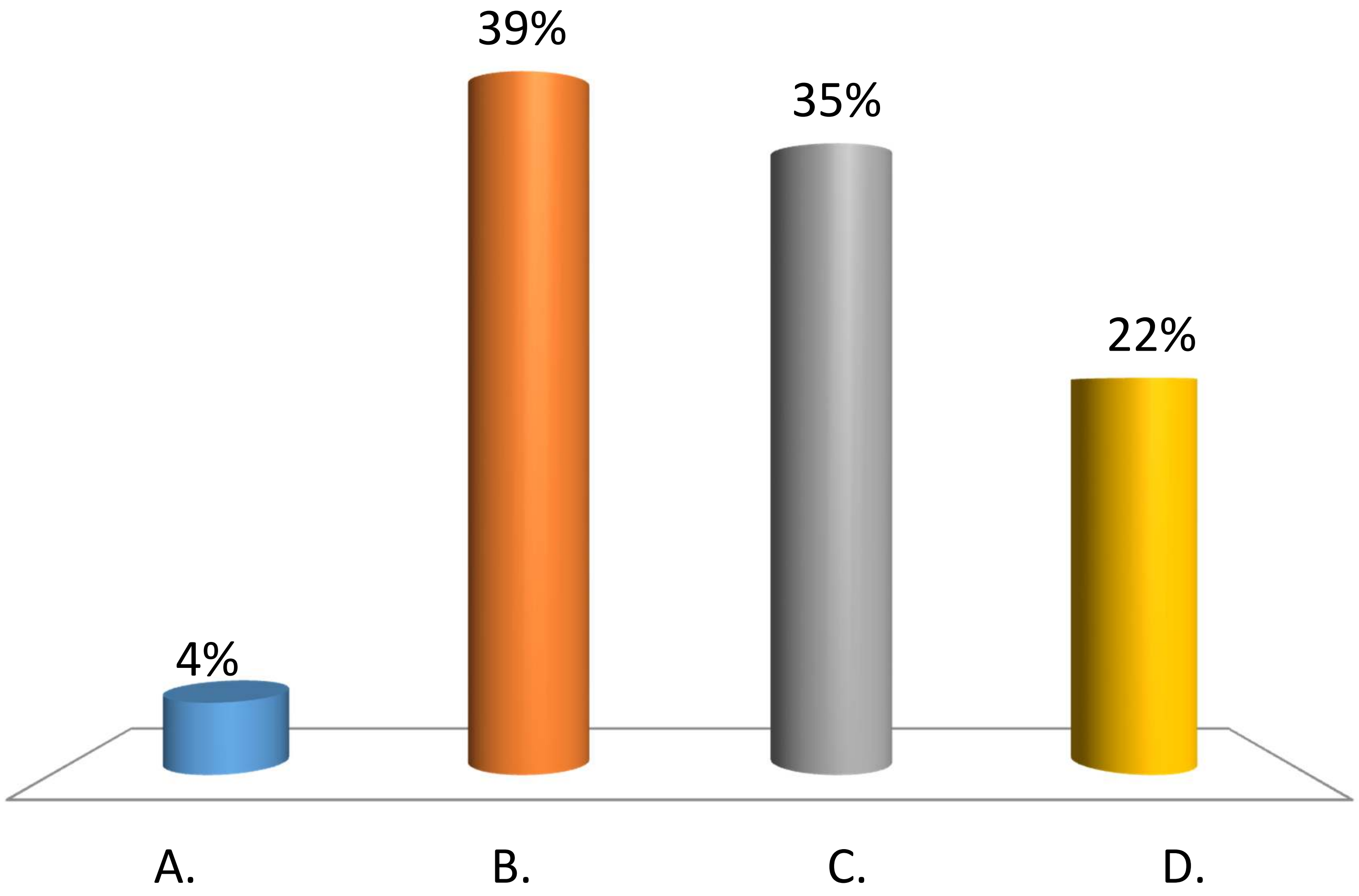


What is the angular speed of a ball rolling w/o slipping?

Ball (radius R) is rolling without slipping



A. Angular speed is zero.
B. Angular speed is $\omega = 2v/R$
C. Angular speed is $\omega = v/R$
D. Angular speed does not depend on speed v



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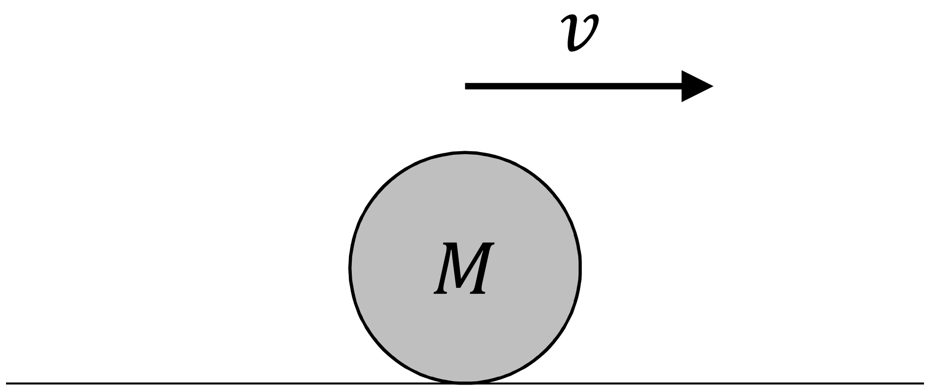


A. Angular speed is zero.

B. Angular speed is $\omega = 2v/R$

C. Angular speed is $\omega = v/R$

D. Angular speed does not depend on speed v

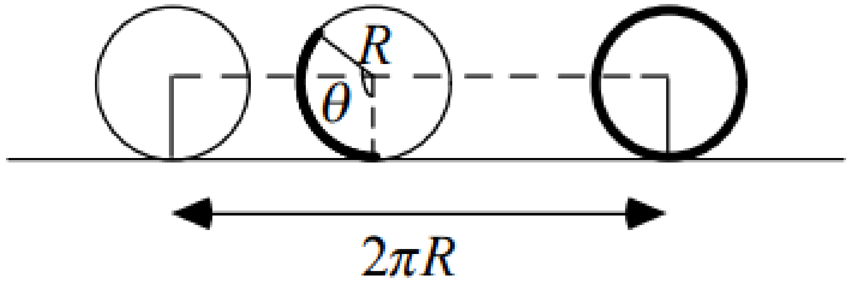
During one revolution, the ball moves a distance

$$\Delta x = 2\pi R$$

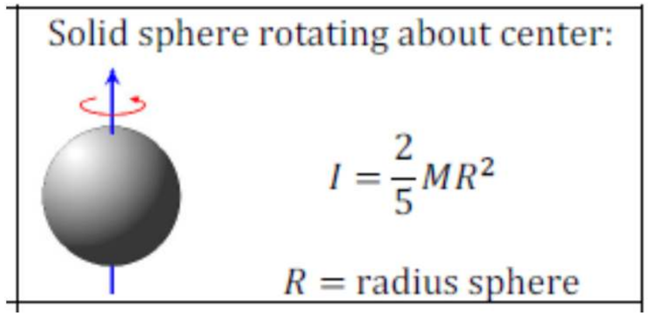
which takes $\Delta t = \Delta x/v = 2\pi R/v$ seconds.

We rotate 2π radians in those Δt seconds, so

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{\Delta t} = \frac{2\pi}{(2\pi R/v)} = \frac{v}{R}$$



What is the total kinetic energy of the rolling ball?



- It is moving at a speed v

- It is rotating at an angular speed $\omega = v/R$

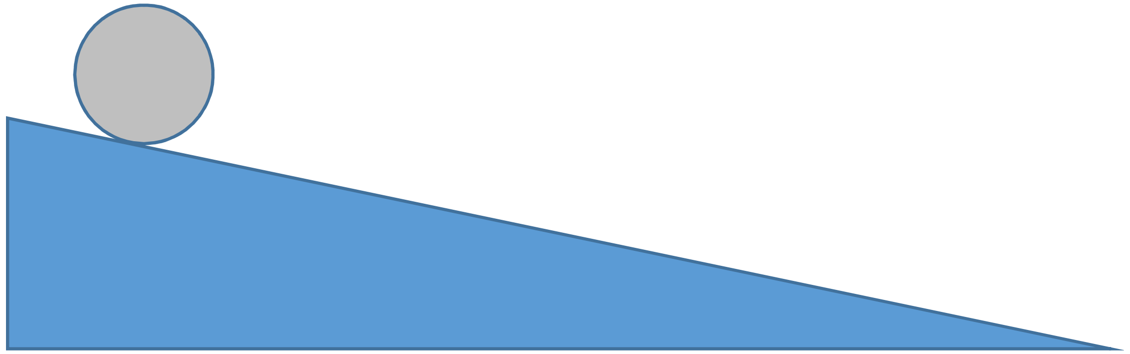
- So total kinetic energy is: $K_{total} = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$

- Since $I = \frac{2}{5}MR^2$ we can simplify this as:

$$K_{total} = \frac{1}{2}Mv^2 + \frac{1}{2}\left(\frac{2}{5}MR^2\right)(v/R)^2 = \frac{1}{2}Mv^2 + \frac{2}{10}Mv^2 = \frac{7}{10}Mv^2$$

What is the angular speed of a ball rolling w/o slipping?

Three objects are rolling down.



- I've got 3 objects with the same mass

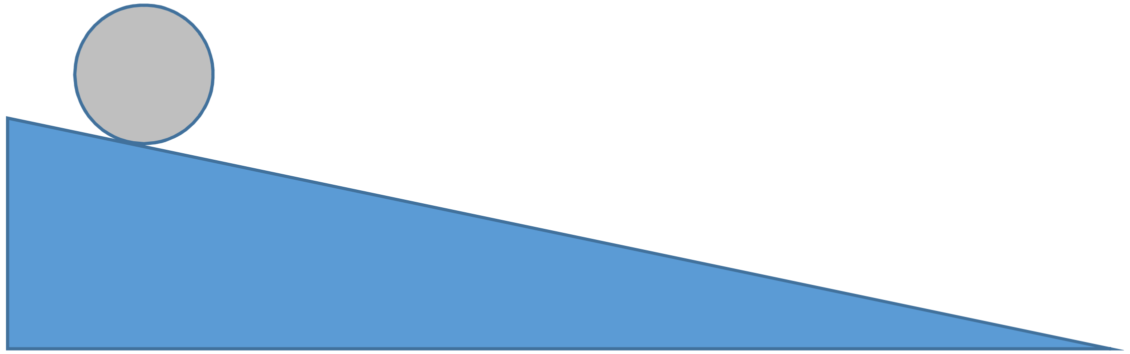
- Hollow cylinder
- Solid cylinder
- Solid sphere

- I will let them go at the same time, which object hits the bottom first?

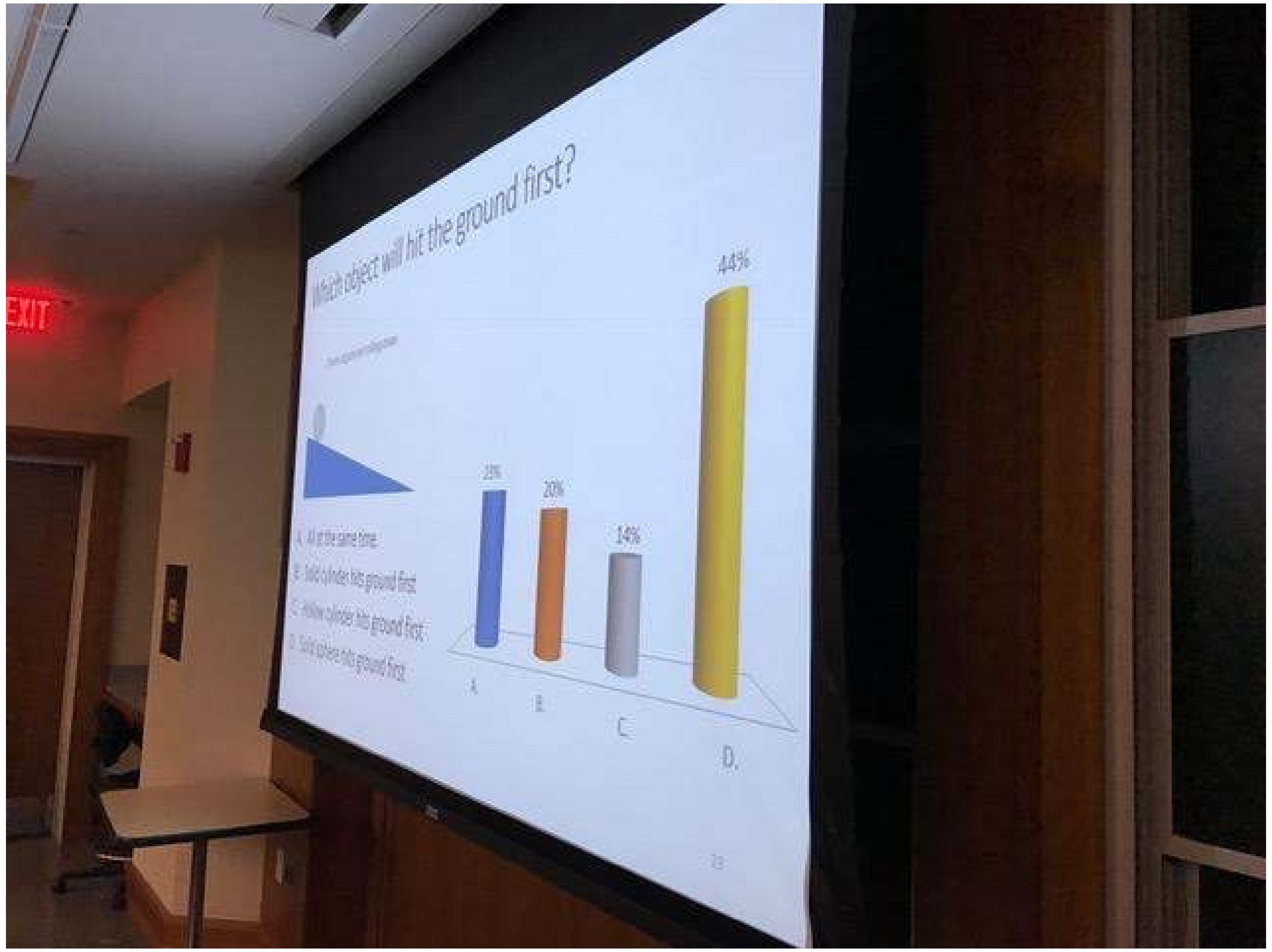
(this will be a clicker question!)

Which object will hit the ground first?

Three objects are rolling down.



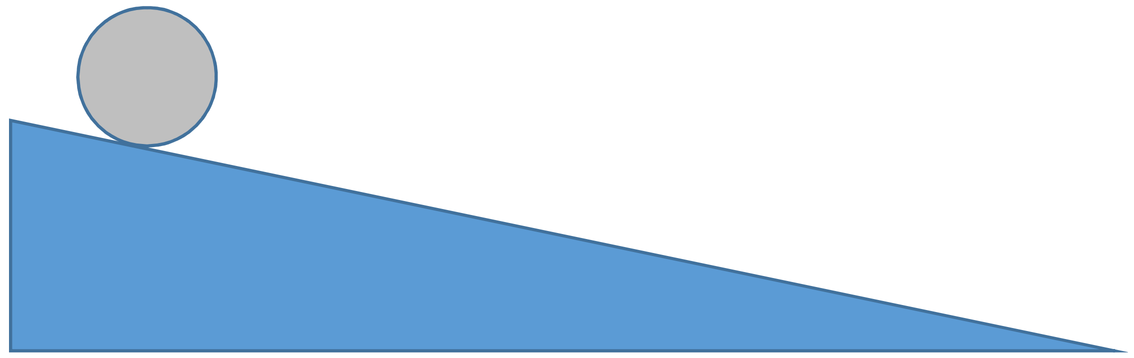
A. All at the same time.
B. Solid cylinder hits ground first
C. Hollow cylinder hits ground first
D. Solid sphere hits ground first



Before I mention that
"oh, the objects have the same mass!"

Which object will hit the ground first?

Three objects are rolling down.
All of them have the same mass!



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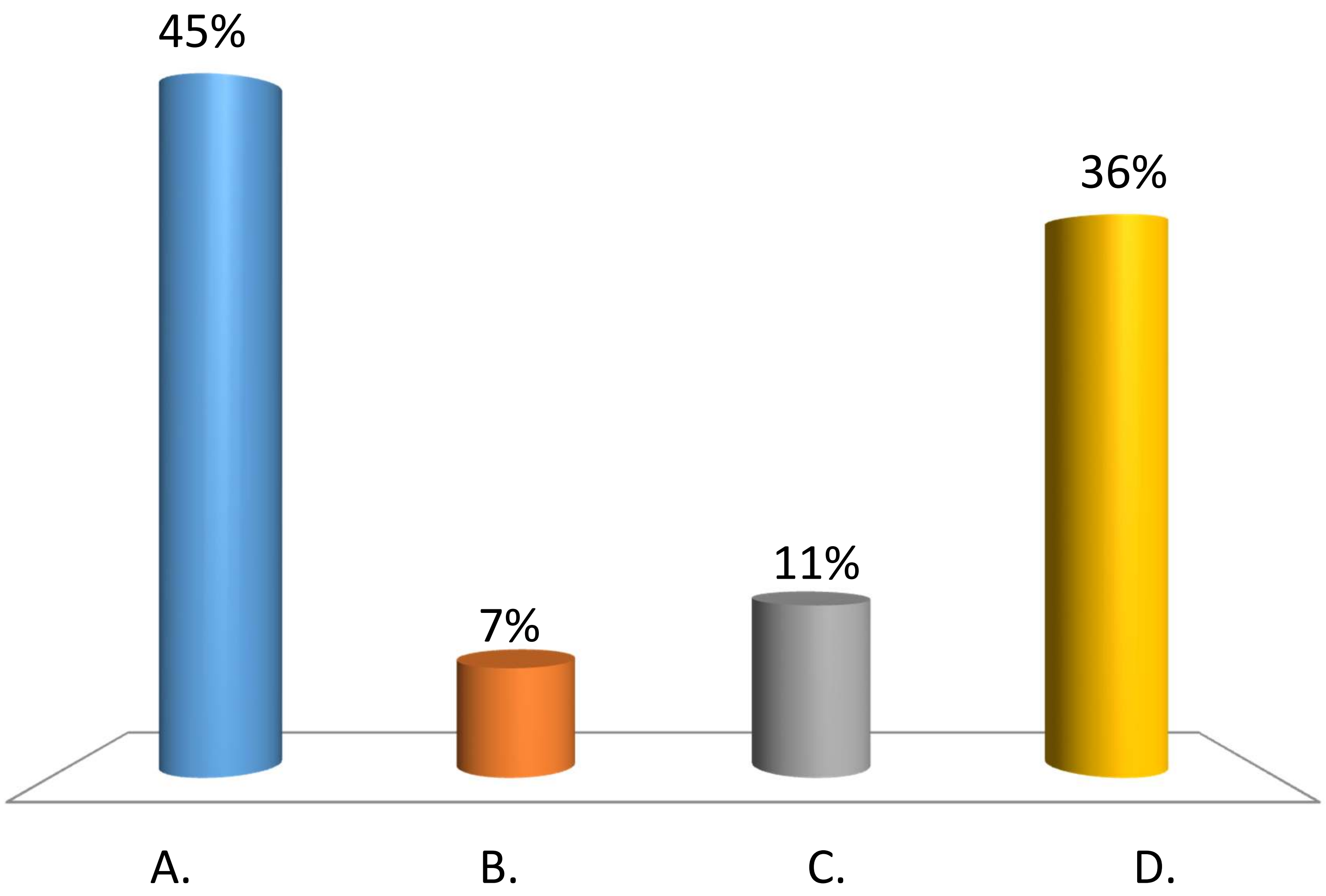

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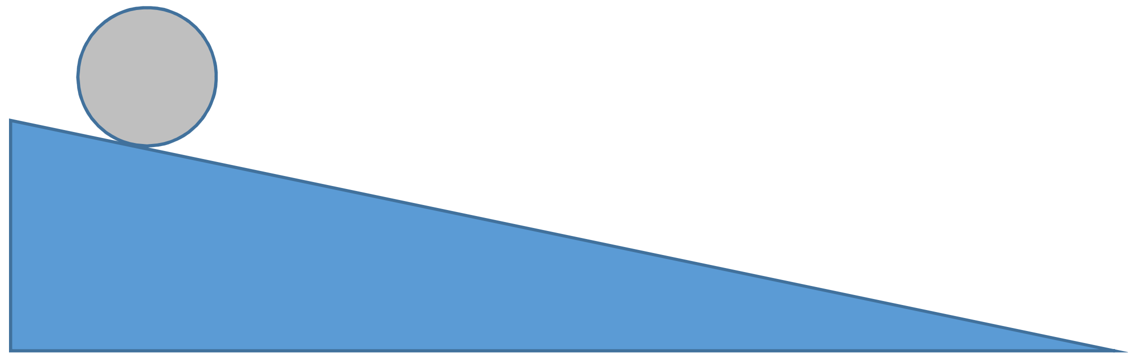

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NOTE: I'm just using my physics intuition,
I haven't tested it yet!

Which object will hit the ground first?

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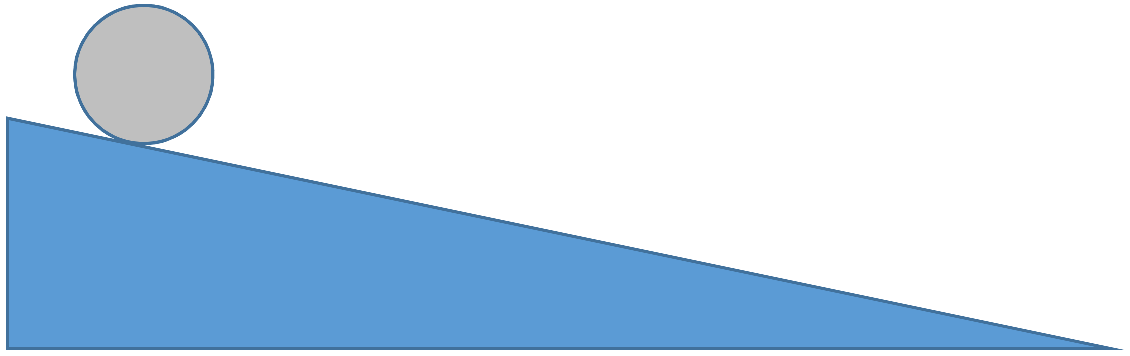

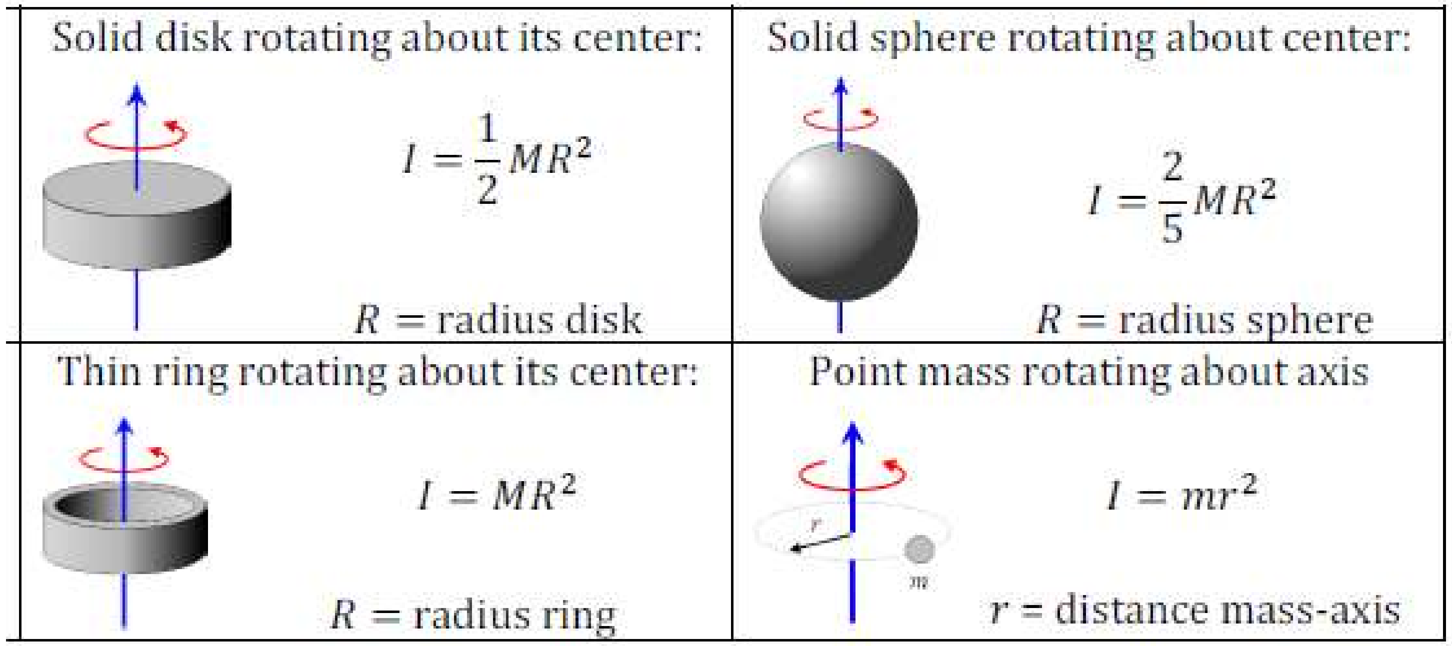

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2/5 = 0.4 < 1/2 so sphere has smallest I

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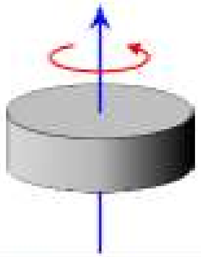
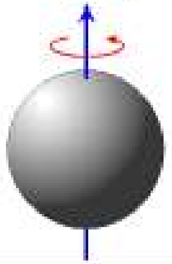
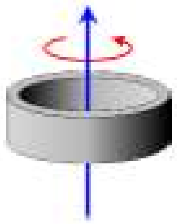
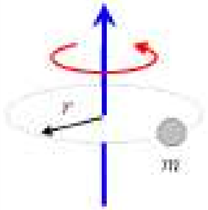
Which object will move faster when it hits bottom?

Solid disk rotating about its center:  $I = \frac{1}{2}MR^2$ R = radius disk	Solid sphere rotating about center:  $I = \frac{2}{5}MR^2$ R = radius sphere
Thin ring rotating about its center:  $I = MR^2$ R = radius ring	Point mass rotating about axis  $I = mr^2$ r = distance mass-axis

2/5 = 0.4 < 1/2 so sphere has smaller I

You should use $mgh = K_f$ to solve this!
But use the TOTAL kinetic energy

Angular momentum

Angular momentum, L

- Similar to "translational" momentum $p = mv$ but for rotations.
- Equation is: $L = I\omega$
- If there is no external torque, angular momentum is conserved: $L_i = L_f$

Conservation of angular momentum

- If there is no external torque, we have: $L_i = L_f$

- How to explain?

Conservation of angular momentum

- If there is no external torque, we have: $L_i = L_f$

- How to explain?

With some demos!

Thank you!

See you next time: Friday 31 March at 2:30pm