Lecture #2 Crystal Structure of Solids

100 years ago
... were just beginning to understand the atom
... had very little idea about how atoms were arranged in a solid
... had no practical use for semiconductors
... had no conceptualization for how electrons (e^-s) move through a solid

Now? we have this class!!

Semiconductors

periodic table:

	III	IV	V
	5 B		
	13 Al	14 Si	15 P
	31 Ga	32 Ge	33 As
	49 In		51 Sb

Silicon: Foremost semiconductor thanks to:
→ abundance
→ native oxide (SiO_2)

Germanium: other elemental semiconductor

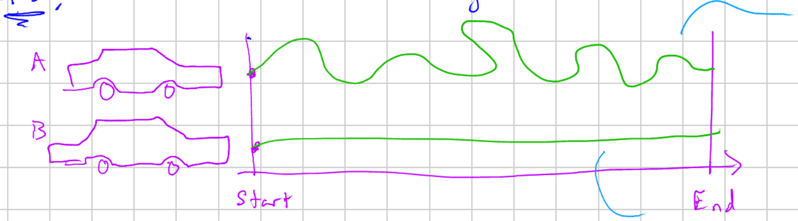
- Dopants used to modulate the conductivity of a semiconductor
- compound semiconductors, or "III-V s" (three-fives)
  e.g., GaAs, InP, AlGaAs, etc.

Solids

How do the atoms arrange themselves within a solid semiconductor?

- Does it matter?

YES! Think of 2 cars racing:



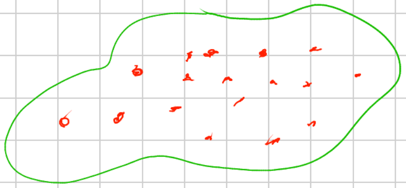
difficult to predict travel time, speed, etc.

predictable speed, travel time

- Three options:

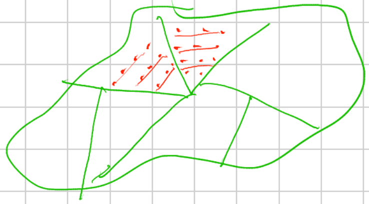
AMORPHOUS

No recognizeable long-range order



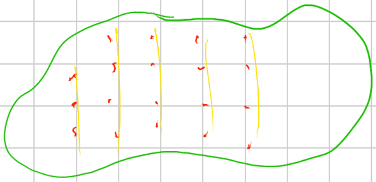
POLYCRYSTALLINE

Completely ordered in segments



SINGLE CRYSTAL

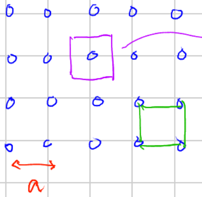
Entire solid is made up of atoms in ordered array



Crystal Structure (lattice)

- unit cell:

2D lattice



unit cell option 1

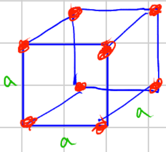
unit cell option 2

lattice constant (atomic spacing)

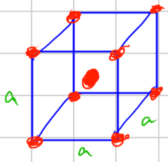
Both work and both have the same # atoms within (ONE!)

- cubic unit cells

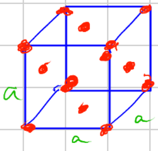
most metals have these lattices



simple cubic



Body-centered cubic



Face-centered cubic (FCC)

- Volume density of atoms in a crystal

Ex. FCC with $a = 4.25\ \text{Å}$ — $\text{Å} = 10^{-10}\,m = 10^{-8}\,cm$

a) # atoms per unit cell?

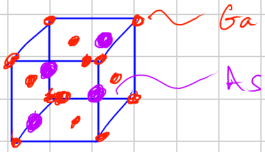
$$\underbrace{8\left(\tfrac{1}{8}\right)}_{\text{corners}} + \underbrace{6\left(\tfrac{1}{2}\right)}_{\text{faces}} = 4$$

b) volume density of atoms

$$V_D = \frac{\#\ \text{atoms / unit cell}}{\text{Volume unit cell}} = \frac{4}{a^3} = \frac{4}{(4.25\times10^{-8}\,cm)^3} = 5.71\times10^{22}\ \frac{\text{atoms}}{cm^3}$$

- Semiconductor unit cells
  - Diamond lattice – FCC lattice with additional FCC included with a $\left(\frac{a}{4}, \frac{a}{4}, \frac{a}{4}\right)$ shift
  - Zincblende lattice – Same, but 4 additional atoms come from other element

ex: GaAs

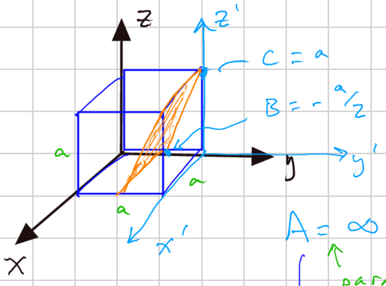


(If Si, all atoms are same)

Si: $a = 5.43\ \text{Å}$

→ Miller Indices (crystallographic planes) – orientation of certain cuts, or planes, in a crystal

(h k l)



How to determine Miller Indices:

① Intercepts to x, y, z axes: A, B, C

If not intersecting axes, then move coord. system (origin) but be sure it does not pass through the origin

the plane

$A = \infty$ ← parallel to axis

② Reciprocate and multiply by integer if needed:

$$h = \frac{na}{A} \qquad k = \frac{na}{B} \qquad l = \frac{na}{C}$$, n = integer multiple

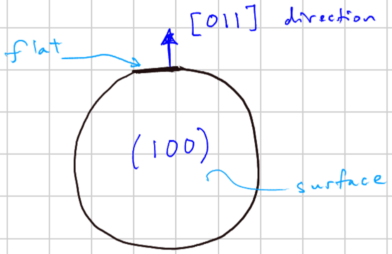
③ If an indice is negative then use bar above #, NOT a minus sign!!

$$h = \frac{a}{\infty} \qquad k = \frac{a}{-a/2} \qquad l = \frac{a}{a}$$

$$0 \qquad\qquad -2 \qquad\qquad 1$$

$$(0\,\bar{2}\,1)$$

- Silicon wafer example:



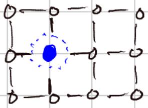
(Demo dicing wafer)

Imperfections and Impurities in Crystals

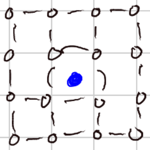
Many different types, but most relevant for semiconductors are:

① vacancies

or substitutional impurities → ☆ Dopants in semiconductors



② Interstitials

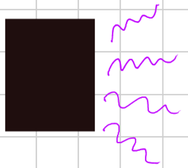


Lecture #3 Quantum Theory of Solids (band structure)

First, reminder of some key aspects of atoms learned over time:

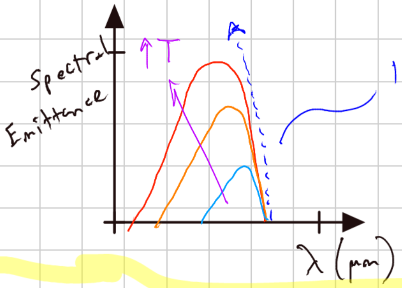
① Black body Radiation

↖ an ideal radiator



When heated (above 0 K), emits radiation

→ everything does!!



19th century models good at long λ, but not at short λ (∞ energy!)

- Max Planck came up with answer (1901)
  - → vibrating atoms can only radiate (or absorb) energy in discrete (quantum) packets
  - → For oscillating frequency ν, energy is to be restricted to:

$$E_n = n h\nu = n\hbar\omega$$

quantum number $n = 0, 1, 2, \ldots$

Planck's constant $= 6.63\times10^{-34}$ J·s

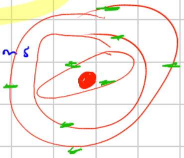
$\hbar = h/2\pi$

Take away?

✳ In atomic scale systems, quantized (or discrete) allowed energy exists, rather than a continuum

② Bohr atom

e^-s in discrete energy levels in atoms



③ Wave-particle duality

Einstein postulated in 1905 to explain photoelectric effect

→ Energy of impinging light $= E = h\nu = mc^2$

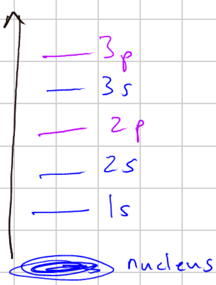
c = speed of light; m = "mass" of a photon

→ momentum: $\boxed{p = \frac{h}{\lambda}} = mc$

λ = wavelength → the de Broglie hypothesis: wave-like nature to matter!

④ Wave-mechanical atom model

e^-s fill orbitals defined by quantum numbers



orbitals n, subshells ℓ, magnetic m_ℓ, and spin m_s

Quantum Mechanics

→ Needed explanation of how exp't data was at odds with classical mechanics / laws of physics

→ 1926, Schrödinger developed wave mechanics for describing micro & nano universes

→ ~same time, Heisenberg developed matrix mechanics → combined and called: quantum mechanics

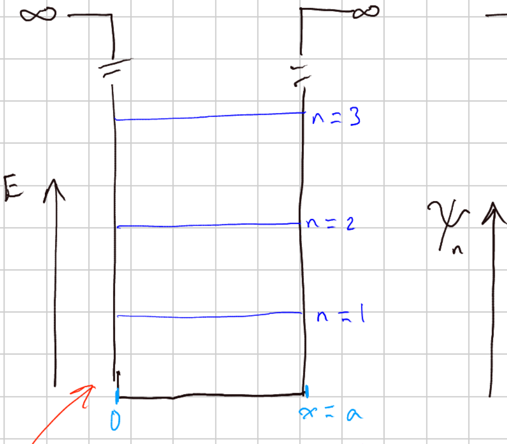
much more difficult math but proved to be equivalent!

Quantum Mechanics

Schrödingers Equation

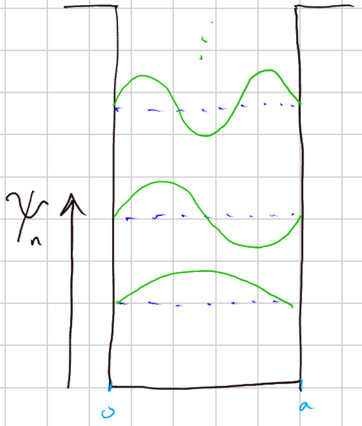
principle governing equation: $$-\frac{\hbar^2}{2\underbrace{m}_{\text{mass}}}\nabla^2\underbrace{\Psi}_{\text{wave function}} + \underbrace{U(x,y,z)}_{\text{potential energy}}\Psi = -\frac{\hbar}{i}\frac{\partial\Psi}{\partial t}$$

- most basically applied to a "particle in a box" (1D)

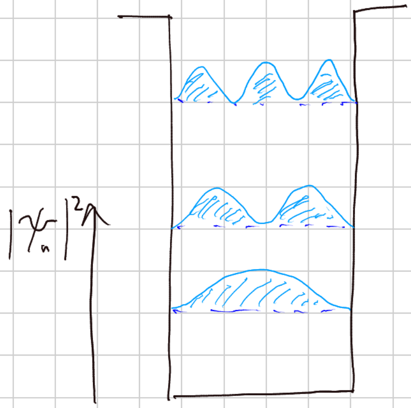


Allowed Energy States/levels

Look familiar? Same concept as e⁻s bound in atoms



Wave functions at the first 3 discrete energy levels



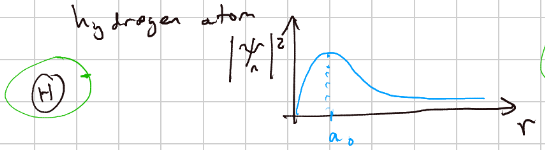
Proportional to the probability of finding the e⁻/particle at a certain point in the well

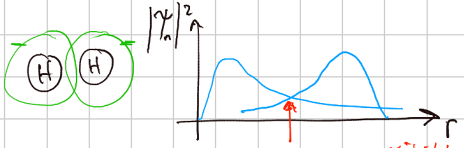
Energy Bands

What happens when atoms are brought close enough together to have their e⁻ orbitals interact? (overlap)

⟹ "Energy bands" are formed by the splitting of energy levels

- Example: hydrogen atom

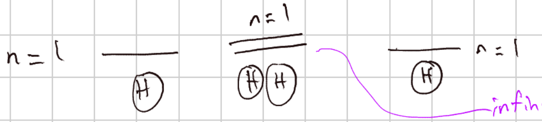




non zero possibility of 2 e⁻s occupying same space

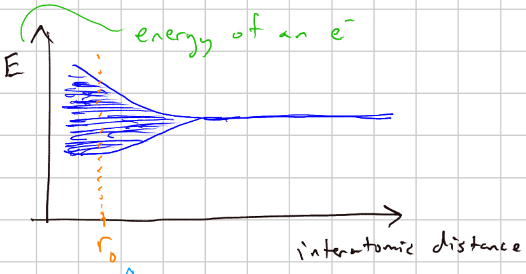
⟹ Pauli exclusion principle says: NO

What happens?



infinitesimally small E between them

Summary:



NOTE: This will happen for each e⁻ orbital, forming "bands" of allowed energy throughout crystal instead of orbitals in atoms

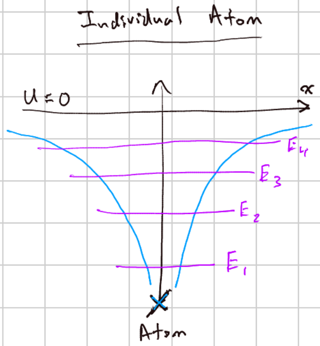
Lecture #4

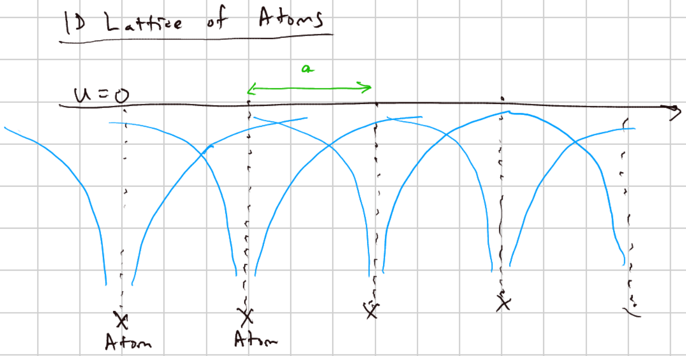
Quantum Theory of Solids (electron transport)

Kronig-Penney Model

Energy bands tell us where e^- are allowed to be, but what dictates how they move from one place to another?

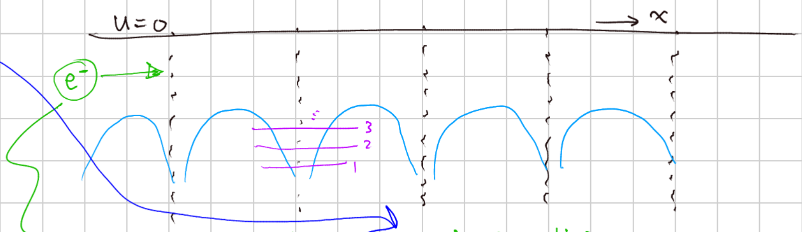
→ most influential is their attraction to the atomic nuclei


Individual Atom


1D Lattice of Atoms

☆ Kronig-Penney model approximates with square step functions and solves Schrödinger's equ.

Net potential profile:



e^- moving through lattice experiencing this PERIODIC potential energy

What's the result?

- A solution for energy of an e^- with relation to K
- This solution/relation between E and k is also known as a "dispersion relation"

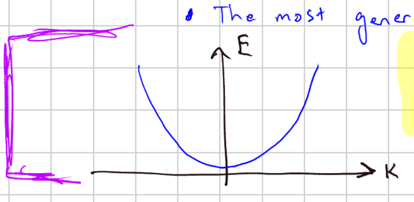
"wave number" and the momentum (p) of the e^- is: $\boxed{p = \hbar k}$, $E = \frac{p^2}{2m}$

- The most generalized solution gives this relation for energy:

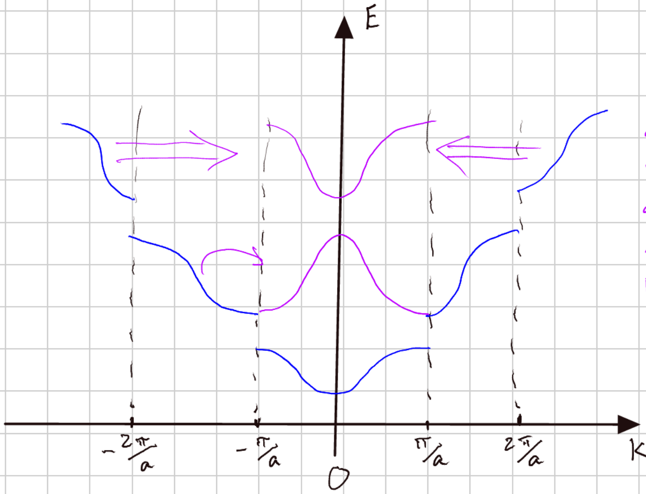
$$\boxed{E = \frac{\hbar^2 k^2}{2m}}$$

parabolic dispersion relation for "free particles" (NOT in 1D periodic lattice)

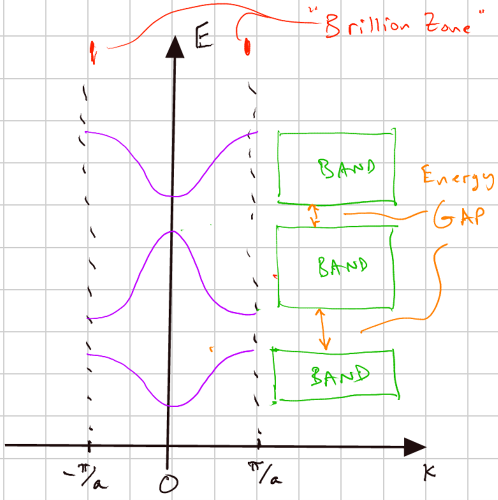
Side point: free particle



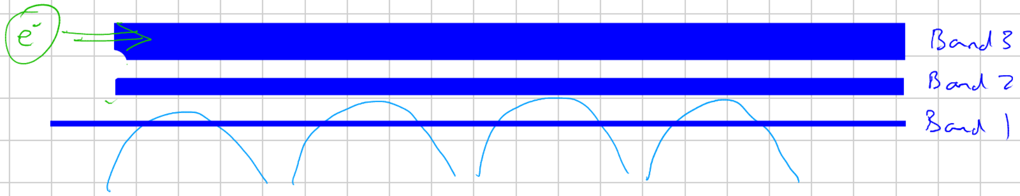
If the periodic conditions of the crystal are applied, then the "allowed" values of k for which the wave equation has a solution give you your E-k diagram:



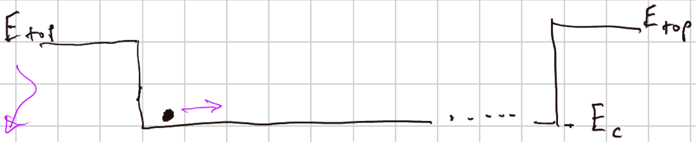
Due to periodicity of 2π, can translate these allowed energy states to a reduced k-space representation.



Visualizing these energy bands in a crystal:



generally, an e⁻ will be in an "upper" or higher energy band and thus is effectively like a particle-in-a-box with:



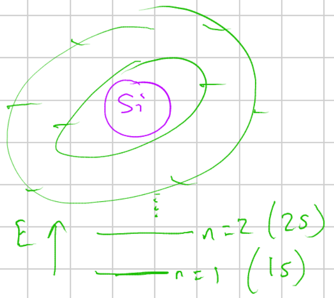
effectively a "free particle" in a well within periodic B.C.

$$E = \frac{\hbar^2 k^2}{2m}, \quad k = \sqrt{\frac{2mE}{\hbar^2}}$$

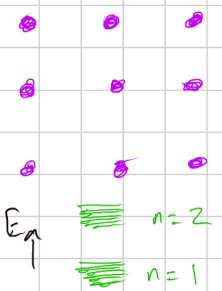
comes from time-independent S.E.:

$$\frac{\partial^2 \psi}{\partial x^2} + \underbrace{\frac{2mE}{\hbar^2}}_{k^2}\psi = 0$$

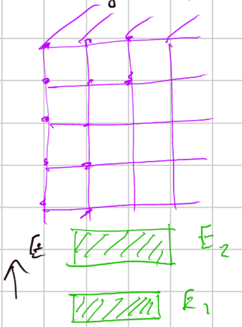
- Where are e⁻s allowed to be in a semiconductor crystal?



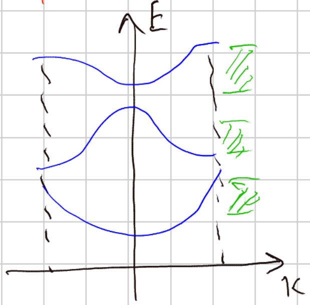
Atoms: e⁻s in orbitals



N atoms brought together causes E-levels to split N times



Energy Bands formed in crystals



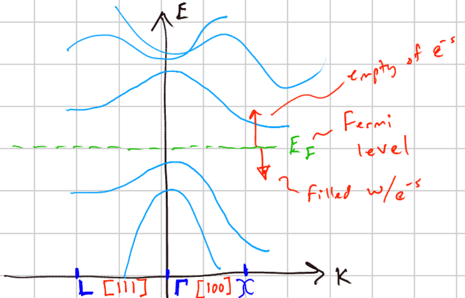
Solving wave eqn. in periodic crystal yields relationship between E-K

Wave number
✱ (crystallographic direction vector)

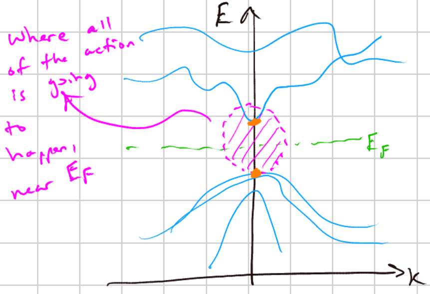
- e⁻ can only be on a line in K-space, hence a change in E is a movement through K-space or "momentum space"

- Thus far this was all developed for 1D, how does it look in 3D crystal?

E-K diagram for Si



E-K for GaAs



★ These are the +K for each with $\Gamma : K = 0$ since the −K is just a mirror image of +K

- What properties does an e^- have?
  - → **Effective mass** (m^*) Consider parabolic E−K relation: $E = \frac{\hbar^2 K^2}{2m}$

$$\frac{dE}{dK} = \frac{\hbar^2 K}{m} = \frac{\hbar p}{m} \quad (p = \hbar K)$$

Velocity

relate p to velocity by $v = \frac{p}{m} = \boxed{\frac{1}{\hbar}\frac{dE}{dK}} = \frac{p}{m} = v$

take 2nd derivative: $\frac{d^2E}{dK^2} = \frac{\hbar^2}{m} \Rightarrow \boxed{m^* = \hbar^2\left(\frac{d^2E}{dK^2}\right)^{-1}}$

★ For a free e^-, mass is constant, but when bound in a crystal with periodic potential, it depends on inverse of E−K curvature.

Electric field

m^* ties classical world to quantum: $F = ma = -eE$

q

— (See textbook) —

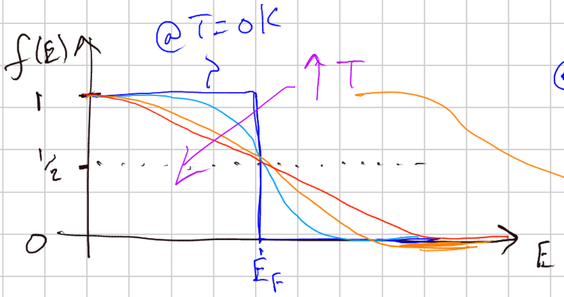
- Where are the e^-s?
  - Fermi-Dirac Probability function — gives the statistical probability of an energy state at energy E being filled with e^-

$$\boxed{f(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{K_B T}\right)}}$$

Fermi level

temperature

Boltzmann constant



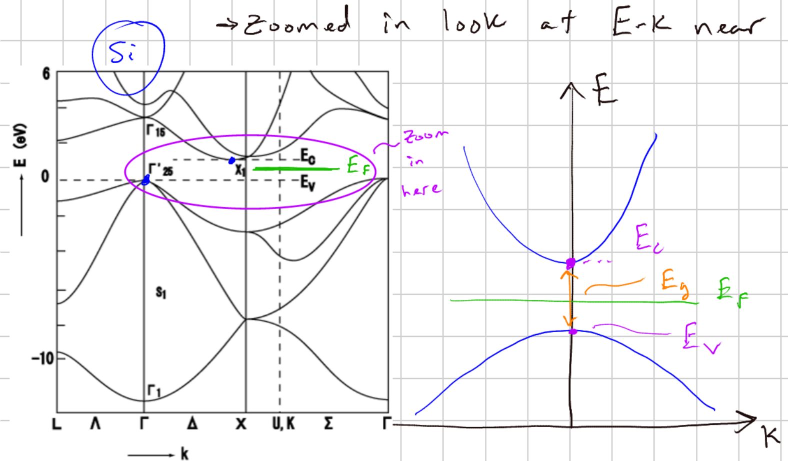
@T=0K, all energy states at $E < E_F$ are filled and all above are empty

→ as T↑, fermi fn. spreads out around E_F

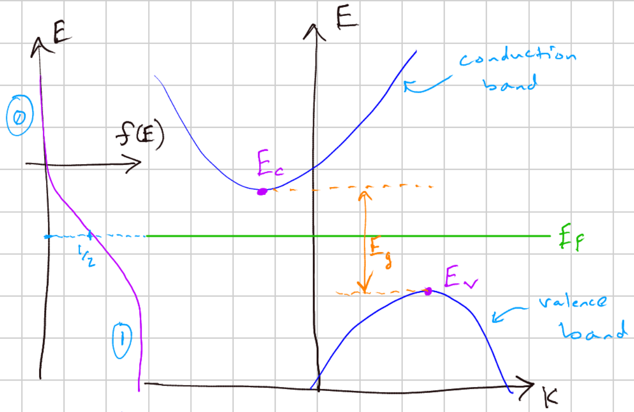
Lecture #5 Thermal Equilibrium

— First: Recap and wrap up discussion of "Where are the e^-s?"

→ Zoomed in look at E-k near E_F:



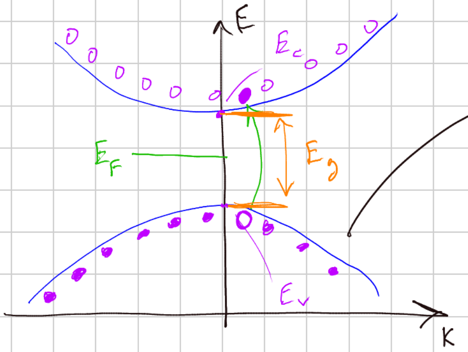
Direct Band Gap Semiconductor



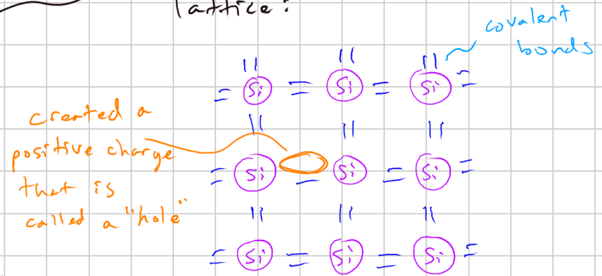
Indirect Band Gap Semiconductor

★ NOTE: The approx. parabolic nature of conduction & valence bands near E_c/E_v allows us to use E-k parabolic expression from free particle.

→ Zoom in further to see if e^- have enough energy to "jump" into conduction Band



What this looks like in the lattice:



- Continuing our "Where are the e^-s?" discussion:

★ For holes, moving ↑E is easier, they have positive charge and their own m_p^*

→ Density of States (DOS)

- How many energy states are there at a certain energy?

★ NOTE: There are several different ways to derive DOS. I will give one different from the textbook so you have options:

- 3D-DOS derivation for parabolic E-k, $E = \frac{\hbar^2 k^2}{2m}$, and crystal that is L in length for x, y, z

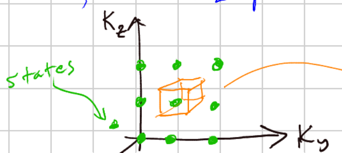
① Determine the # states per unit volume of K-space

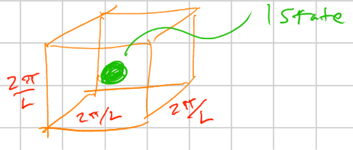
★ From the S.E., allowed wavenumbers in periodic potential are: $K = \frac{2\pi}{L} n$

For this class we will always use this → length of crystal; integer

where volume of the entire crystal is $V = L^3$

- So, with $\frac{2\pi}{L}$ periodicity, a box around one state in k-space:





$$\rightarrow \frac{1\text{ state}}{\left(\frac{2\pi}{L}\right)^3} = \frac{1}{\frac{8\pi^3}{V}} = \frac{V}{8\pi^3} \times 2 = \boxed{\frac{V}{4\pi^3}}$$

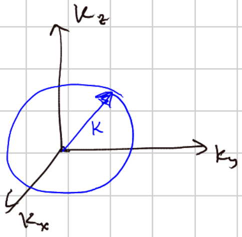
allowed energy states / unit volume k-space

e^- spin ↑, ↓ for each state

② Find total # states up to a value K

$$N(K) = \left(\frac{\text{allowed states}}{\text{unit volume k-space}}\right)\left(\text{total volume up to } K\right)$$

$$N(K) = \left(\frac{V}{4\pi^3}\right)\left(\frac{4}{3}\pi K^3\right) = \boxed{\frac{VK^3}{3\pi^2}}$$



③ Convert to total # states up to certain energy E

use this conversion from E-k : $k = \sqrt{\frac{2mE}{\hbar^2}}$) $E = \frac{\hbar^2 k^2}{2m}$

$$N(E) = \frac{V}{3\pi^2}\left(\sqrt{\frac{2mE}{\hbar^2}}\right)^3 = \frac{V}{3\pi^2\hbar^3}(2mE)^{3/2}$$

④ Differentiate w.r.t. energy and divide by volume for DOS per unit volume:

$$DOS_{3D} = g(E) = \frac{1}{V}\frac{d}{dE}N(E) = \frac{m\sqrt{2mE}}{\pi^2\hbar^3} = g(E)_{3D}$$ ✱

$\hbar = \frac{h}{2\pi}$

$$= \frac{4\pi}{h^3}(2m)^{3/2}\sqrt{E}$$

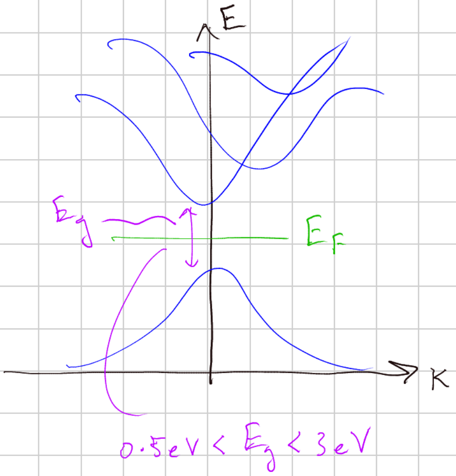
Applying to conduction & valence bands (approx. parabolic) gives:

conduction band : $E = E_c + \frac{\hbar^2 k^2}{2m_n^*} \Rightarrow \boxed{g_c(E) = \frac{4\pi}{h^3}(2m_n^*)^{3/2}\sqrt{E - E_c}}$

Valence band: $E = E_v - \frac{\hbar^2 k^2}{2m_p^*} \Rightarrow \boxed{g_v(E) = \frac{4\pi}{h^3}(2m_p^*)^{3/2}\sqrt{E_v - E}}$

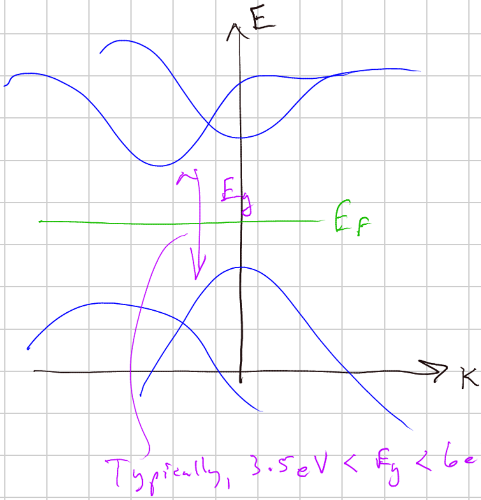
- What makes a crystal a semiconductor?

SEMICONDUCTOR



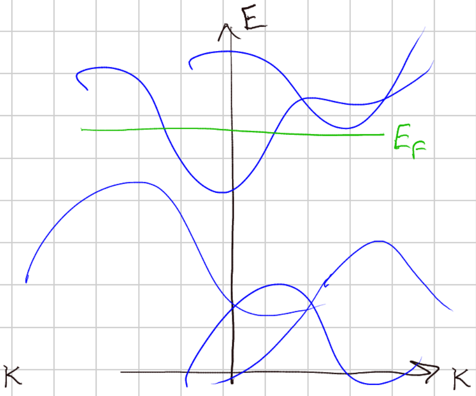
$0.5\text{eV} < E_g < 3\text{eV}$

INSULATOR



Typically, $3.5\text{eV} < E_g < 6\text{eV}$

METAL

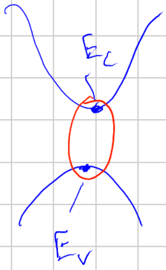


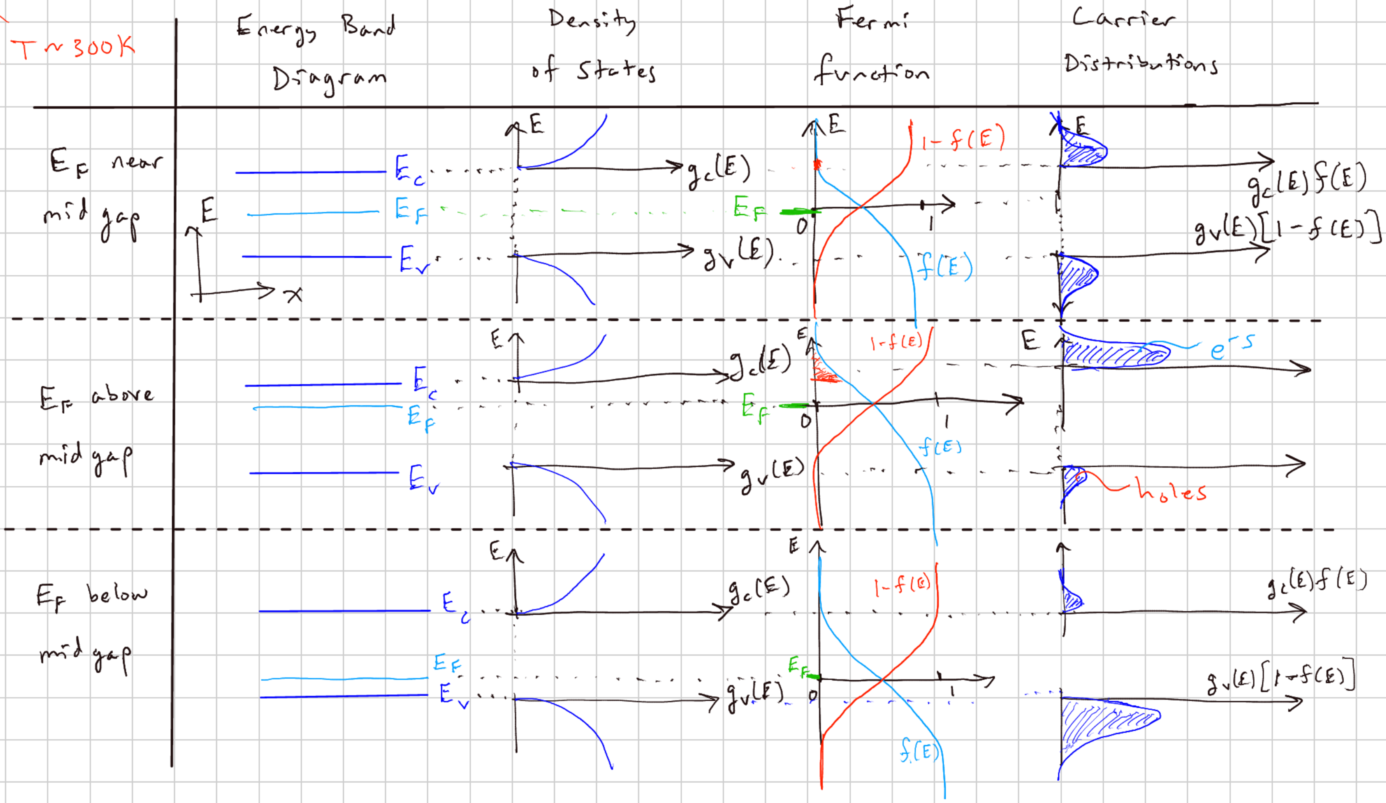
✱ Full or empty bands do not conduct!

Charge Carrier Distribution

Let's start with an overview that we can refer back to:

room temperature @ T ~ 300K





★ Equilibrium (EQ) is when no external forces (E-fields, temperature gradient, etc.) are acting on semiconductor

$\# e^{-}\text{'s}: n(E) = g_c(E) f(E)$ $\# h^{+}\text{'s}: p(E) = g_v(E)[1 - f(E)]$

Intrinsic Semiconductor: Ideally, a semiconductor with NO impurities. E_F is within band gap.

★ → Equal concentration of e^{-}'s and h^{+}'s

$n_i \equiv e^{-}$ concentration

$p_i \equiv h^{+}$ concentration

$\Rightarrow n_i = p_i$

Intrinsic Semiconductor

We just use this since they're equal!

Lecture #6

Thermal Equilibrium (cont)

Equilibrium (EQ) is when no external forces (E-fields, temperature gradient, etc.) are acting on semiconductor

$$\# e^{-}s: n(E) = g_c(E) f(E) \qquad \# h^{+}s: p(E) = g_v(E)[1-f(E)]$$

Intrinsic Semiconductor: Ideally, a semiconductor with NO impurities. E_F is within band gap.

→ Equal concentration of $e^{-}s$ and $h^{+}s$ →

$n_i \equiv e^{-}$ concentration

$p_i \equiv h^{+}$ concentration

$\Rightarrow n_i = p_i$

intrinsic Semiconductor

We just use this since they're equal!

Thermal EQ carrier concentrations:

e^{-} conduction band

$$n_0 = \int_{E_c}^{\infty} g_c(E) f(E)\, dE$$

h^{+} valence band

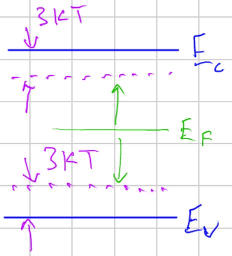
$$p_0 = \int_{-\infty}^{E_v} g_v(E)[1-f(E)]\, dE$$

IF E_F isn't too close to E_c or E_v, then can use Boltzmann approx:

$$f(E) = \frac{1}{1+\exp\left(\frac{E-E_F}{kT}\right)} \approx \exp\left[\frac{-(E-E_F)}{kT}\right]$$

If $E_v + 3kT \leq E_F \leq E_c - 3kT$

visualized:

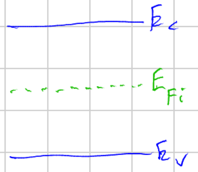


Using the approx gives:

$$n_0 = \underbrace{2\left(\frac{2\pi (m_n^{*}) kT}{h^2}\right)^{3/2}}_{\equiv N_c \text{, effective density of states in conduction band}} \exp\left[\frac{-(E_c-E_F)}{kT}\right]$$

★ Almost always

$E_{Fi} \approx$ mid-gap $= \frac{E_c - E_v}{2}$



intrinsic Fermi level

Then:

Thermal EQ, $E_v + 3kT \leq E_F \leq E_c - 3kT$

$$n_0 = N_c \exp\left[\frac{-(E_c-E_F)}{kT}\right]$$

$$N_c = 2\left(\frac{2\pi m_n^{*} kT}{h^2}\right)^{3/2}$$

$$n_0 = n_i \exp\left[\frac{E_F - E_{Fi}}{kT}\right]$$

$$p_0 = N_v \exp\left[\frac{-(E_F-E_v)}{kT}\right]$$

$$N_v = 2\left(\frac{2\pi m_p^{*} kT}{h^2}\right)^{3/2}$$

$$p_0 = n_i \exp\left[\frac{E_{Fi} - E_F}{kT}\right]$$

Apply these n_o, p_o expressions to the intrinsic S.C. case where:

$$n_o = n_i = p_o$$

and define the E_F for an intrinsic s.c. as: E_{Fi}

☆ NOTE: E_{Fi} is at midgap if $g_c(E) = g_v(E)$ [Symmetric DOS]

No matter what: $E_c - E_v = E_g$ and:

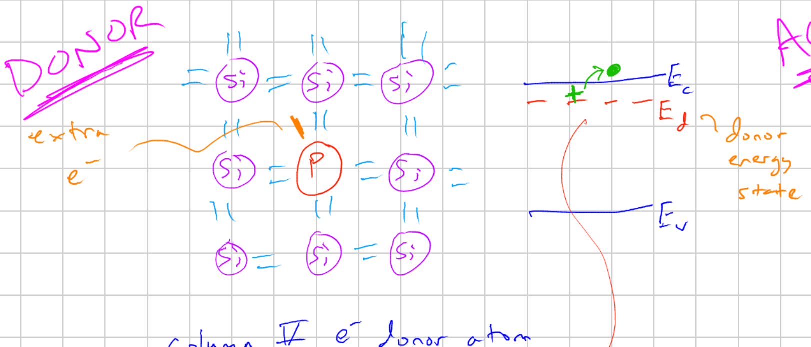
Intrinsic carrier density:

$$\boxed{n_i^2 = N_c N_v \exp\left[\frac{-E_g}{kT}\right]}$$

Thermal EQ ≡ constant, independent of E_F!

Dopant atoms and E_F position

When adding an impurity to a S.C. it can contribute to carrier concentration



column V **e^- donor atom**

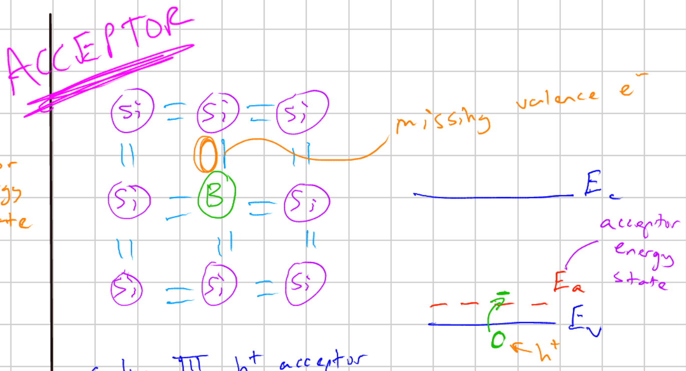
once e^- has sufficient energy to go to E_c, donor is ionized (+)

☆ NOT a h^+, but a fixed + ion

☆ N_d ≡ # donor dopant atoms

"n-type" doping

majority carriers: e^-'s minority carr: h^+'s



column III **h^+ acceptor dopant**

e^- from E_v fills E_a state, creating fixed − ion and leaving behind a h^+

N_a ≡ # acceptor dopant atoms

"p-type" doping

majority carriers: h^+s minority: e^-s

→ Once doped, semiconductor is now extrinsic

→ Ionization energy is the E to ionize dopants

- for Si ~ kT

→ It turns out that the product $n_o p_o$ in extrinsic S.C. is:

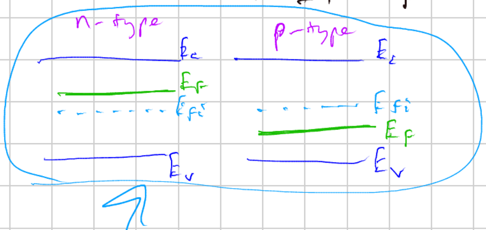
$$n_o p_o = N_c N_v \exp\left[\frac{-E_g}{kT}\right]$$

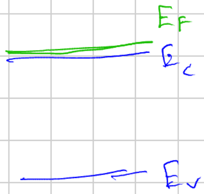
hence: $$\boxed{n_o p_o = n_i^2}$$

☆ Considered a material parameter for given T

☆ product of total carrier conc. at EQ is a constant!

→ If E_F is within E_c or E_v ≡ degenerate s.c.

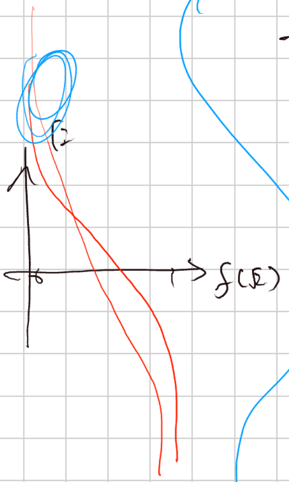


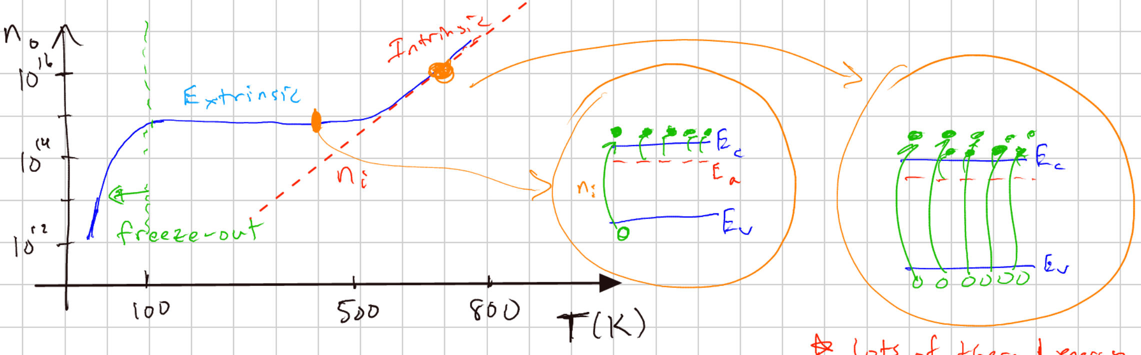


n-type, degenerately doped s.c.

✱ Boltzmann approx. does not hold

→ Temperature (thermal energy) is key to ionizing dopants!





✱ lots of thermal energy for e⁻ jumping E_g

→ E_F position will depend on temperature and doping concentration, N_d, N_a

$$\boxed{E_c - E_F = kT\ln\left(\frac{N_c}{n_o}\right) \cong kT\ln\left(\frac{N_c}{N_d}\right)}$$

- Thermal EQ
- nondegenerate
- n-type
- ~ room temp.

when $N_d >> n_i$

- Charge neutrality – in thermal EQ, s.c. is electrically neutral ≡ net charge density is 0

$$\boxed{n_o + \underbrace{N_a^-}_{\text{ionized acceptors}} = p_o + \underbrace{N_d^+}_{\text{ionized donors}}}$$

assume complete ionization (reasonable for most operating device temps)

$$\Rightarrow \quad n_o + N_a = p_o + N_d \qquad \rightarrow p_o = \frac{n_i^2}{n_o}$$

$$n_o + N_a = \frac{n_i^2}{n_o} + N_d$$

$$n_o^2 - (N_d - N_a)n_o - n_i^2 = 0$$

general case solutions

$$\boxed{n_o = \frac{(N_d - N_a)}{2} + \sqrt{\left(\frac{N_d - N_a}{2}\right)^2 + n_i^2}}$$

- Thermal EQ
- full ionization of dopants

$$\boxed{p_o = \frac{(N_a - N_d)}{2} + \sqrt{\left(\frac{N_a - N_d}{2}\right)^2 + n_i^2} = \frac{n_i^2}{n_o}}$$

☆ Special cases

① Intrinsic Semiconductor $(N_a = 0 \,,\; N_d = 0)$

- $n_o = n_i \quad , \quad p_o = n_i$

② Doped, nondegenerate s.c. with either: $N_d - N_a \simeq N_d >> n_i$ or $N_a - N_d \simeq N_a >> n_i$

→ most typical case found in devices / practical situations

$$n_o \simeq N_d \qquad\qquad p_o \simeq N_a$$
$$\text{OR}$$
$$p_o \simeq \frac{n_i^2}{N_d} \qquad\qquad n_o \simeq \frac{n_i^2}{N_a}$$

③ Doped s.c. with $n_i >> |N_d - N_a|$ — lightly doped

→ at reasonable temps, n_i will exceed N_d or N_a

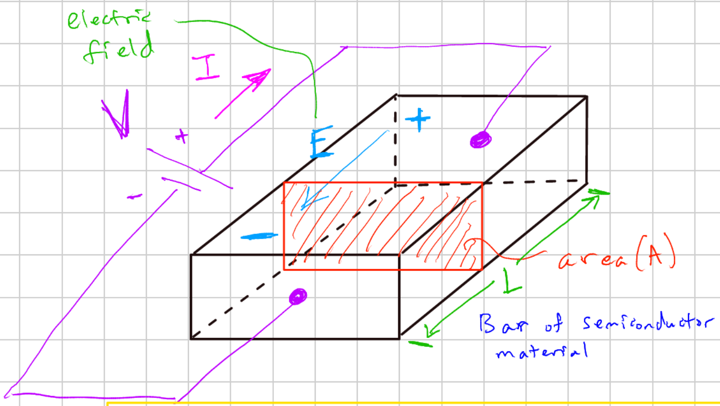
- $n_o \simeq p_o \simeq n_i$

④ Compensated s.c. where N_a and N_d are comparable and nonzero $(N_a - N_d \simeq 0)$

- must use full expressions for n_o, p_o

Lecture #7 Carrier Transport – Drift

Review of some basic electrical behavior:



Ohm's Law: $\underset{\text{voltage (V)}}{V} = \overset{\text{current (A)},\ A = C/s}{I}\,\underset{\text{resistance }(\Omega)}{R}$

current density: $J = \frac{I}{A}$ (A/cm^2) $= \underset{\text{conductivity }(S/cm)}{\sigma}\,\overset{\text{electric field}}{E}$

electric field: $E = \frac{V}{L}$ (V/cm)

resistivity: $\rho = \frac{RA}{L}$ $(\Omega\text{-}cm)$ OR $R = \frac{\rho L}{A}$

conductivity: $\sigma = \frac{1}{\rho}$ (S/cm)

☆ Direction of current flow:
- → h^+ go with E-field, hence J_p goes with E-field
- → e^-s and J_n go against E-field

Drift current – charged carrier motion due to an applied E-field

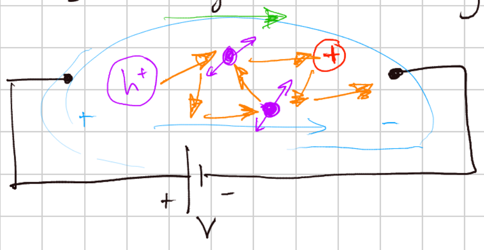
→ Last time we deduced how to determine the # of e^-s (n_o) and h^+s (p_o) and now need to see how they move when an E-field is applied

e^- current $\qquad\qquad$ h^+ current

$$J_{n|drift} = \overset{\text{negative due to } e^- \text{ are neg. carriers}}{-}\underset{\text{elementary charge (q)}}{e}\,n_o\,\overset{\text{average drift velocity}}{v_{dn}} \qquad J_{p|drift} = e\,p_o\,v_{dp}$$

side note

Why "average" drift velocity?



$F = m^* a = eE$

☆ carriers will scatter as they are accelerated by applied E-field:

Lattice scattering: thermal energy causes lattice atoms to vibrate, disrupting the potential profile.

Impurity scattering: ionized impurities disrupt periodic potential

$$v_{dn} = -\overset{e^-\text{ mobility}}{\mu_n}E \qquad v_{dp} = \overset{h^+\text{ mobility}}{\mu_p}E$$

☆ mobility defines how well a carrier will move through a crystal at a given T and $N_{a,d}$ units: $cm^2/V\cdot s$

$$J_{n|drift} = -en_o(-\mu_n E) = e\mu_n n_o E \qquad J_{p|drift} = e\mu_p p_o E$$

$$\boxed{J_{drift} = e(\mu_n n_o + \mu_p p_o)E}$$ total drift current density

Mobility

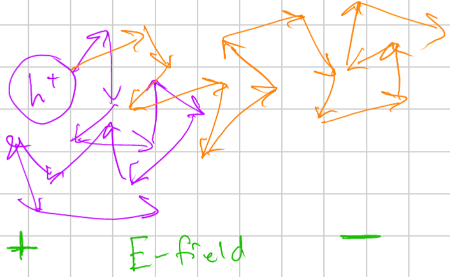
→ one of the most quoted properties of a semiconductor material to indicate material quality for semiconductor devices

Looking back at the impact of E-field on carriers:

$$F = m^*_{c,p} \frac{dv}{dt} = eE$$

assume m^*_c and E are constant: $v = \frac{eEt}{m^*_{c,p}}$

Note on carrier motion: most is thermal (from $T > 0K$) and small E-field just adds some directionality:



⇒ added some net direction/flow to the random thermal motion

$$v = \left(\frac{e\tau_{cp}}{m^*_{c,p}}\right)E$$

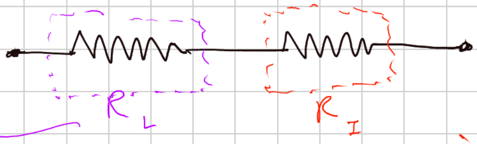
mean time between scattering events/collisions

✭ use a statistical model to determine impact of collisions and their frequency:

$$v = \mu_p E \Rightarrow \boxed{\mu_p = \frac{e\tau_{cp}}{m^*_{cp}}} \text{ or } \boxed{\mu_n = \frac{e\tau_{cn}}{m^*_{cn}}}$$

→ Two scattering events that degrade μ:

Think of them as added resistance to current flow (just an analogy)



✭ impurities more effectively scatter slower carriers

Lattice scattering
- No doping dependence
- Decreases with ↓T

impurity scattering
- Increases with ↑$N_{a,d}$
- Increase with ↓T

"total" mobility: $\frac{1}{\mu} = \frac{1}{\mu_L} + \frac{1}{\mu_I}$

Conductivity

$$J_{drift} = e(\mu_n n_0 + \mu_p p_0)E = \sigma E$$

$$\boxed{\sigma = e(\mu_n n_0 + \mu_p p_0)} \quad S/cm$$

Resistivity

$$\boxed{\rho = \frac{1}{\sigma} = \frac{1}{e(\mu_n n_0 + \mu_p p_0)}}$$ $\Omega \cdot cm$

✻ has dependence on $N_{a,d}$

unique cases:

$N_a >> n_i$, $N_d = 0$

$\sigma \cong e\mu_p p_0$

also, assume complete ionization:

$\sigma = e\mu_p N_a$

Velocity saturation

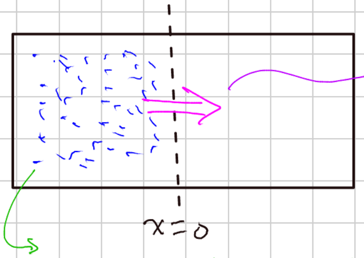
at low-to-moderate E-fields, v_{drift} is linearly ∝ E

✻ E-field does not substantially impact e^- energy but at higher E it does lead to added scattering effects

for holes: $$v_d = \frac{\mu E}{\left[1 + \left(\frac{\mu E}{v_{sat}}\right)\right]} = \begin{cases} \mu E \ \ldots \ E \to 0 \ \text{(small E-field)} \\ v_{sat} \ \ldots \ E \to \infty \ \text{(high E-field)} \end{cases}$$

Carrier Diffusion

consider the example of a container of gas molecules:



they will "diffuse" based on their concentration gradient

have some thermal velocity

→ Same thing applies to carriers in a s.c.

e^- diffusion

$$J_{n|diff} = e v_{th} \ell \frac{dn}{dx}$$

thermal velocity

length < mfp

$$\boxed{J_{n|diff} = eD_n \frac{dn}{dx}}$$

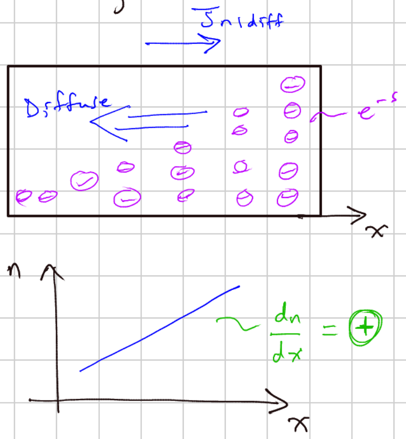
h^+ diffusion

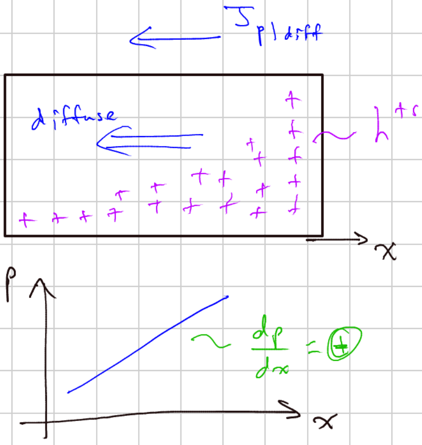
$$J_{p|diff} = -e v_{th} \ell \frac{dp}{dx}$$

$$\boxed{J_{p|diff} = -eD_p \frac{dp}{dx}}$$

diffusion coefficient

Directionality :





Total Current

Summation of drift and diffusion currents for e⁻s & h⁺s

$$J_{total} = J_{n|drift} + J_{p|drift} + J_{n|diff} + J_{p|diff}$$

$$\boxed{J = e n \mu_n E + e p \mu_p E + e D_n \frac{dn}{dx} - e D_p \frac{dp}{dx}}$$