School of Engineering & Applied Science
Ahmedabad University
MA202-Probability and Random processes
Homework Assignment - 1

Submission Date : 3rd February 2017

1. There are n persons in a room.
   (a) What is the probability that at least two persons have the same birthday?
   (b) Calculate this probability for n = 50.
   (c) How large need 'n' be for this probability to be greater than 0.5?

2. Develop a careful proof of Theorem 2.1 which states that for any events A and B,

   $$Pr(A \bigcup B) = Pr(A) + Pr(B) - Pr(A \bigcap B)$$

   One way to approach this proof is to start by showing that the set $A \bigcup B$ can be written as the union of three mutually exclusive sets,

   $$\text{A}\cup\text{B} = \{A \cap \overline{(A \cap B)} \cup \{A \cap B\} \cup \{B \cap \overline{(A \cap B)}\}$$

   And hence by corollary 2.1

   $$\text{Pr}(\text{A}\cup B) = \text{Pr}(A \cap \overline{(A \cap B)}) + \text{Pr}(A \cap B) + \text{Pr}(B \cap \overline{(A \cap B)})$$

   Next,show that

   $$\text{Pr}(A \cap \overline{(A \cap B)}) = \text{Pr(A)-Pr(A}\cap\text{B)}$$

   and likewise

   $$\text{Pr}(B \cap \overline{(A \cap B)}) = \text{Pr(B)-Pr(A}\cap\text{B).}$$

   Hint: Recall DeMorgans law. Put these results together to complete the desired proof.

3. Prove that if Pr(B|A) = Pr(B), then it follows that

   (a) Pr(A,B)=Pr(A)Pr(B), and

   (b) Pr(A|B) = Pr(A)
   Furthermore show that if Pr(B|A) $\neq$ Pr(B), then the two conditions (a) and (b) do not hold as well.

4. (a) Demonstrate that the relative frequency approach to assigning probabilities satisfies the three axioms of probability.
   (b) Demonstrate that the definition of conditional probability
   Pr (A|B) = Pr(A,B)/ Pr(B) satisfies the three axioms of probability.

5. I deal myself 3 cards for a standard 52-card deck. Find the probabilities of each of the following events:

   (a) 2 of a kind (e.g. 2 fives or 2 kings)
   (b) 3 of a kind
   (c) 3 cards all of the same suit (a.k.a flush , e.g. 3 hearts or 3 clubs)
   (d) 3 cards in consecutive order (a.k.a. a straight e.g. 2-3-4 or 10-J-Q)

6. Two six-sided (balanced) dice are thrown. Find the probabilities of each of the following events:

   (a) only 2, 3, or 4 appear on both dice;
   (b) the value of the second roll subtracted from the value of the first roll is 2;
   (c) the sum is 10 given that one roll is 6;
   (d) the sum is 7 or 8 given that one roll is 5;
   (e) One roll is a 4 given that the sum is 7.

7. A computer memory has the capability of storing 106 words. Due to outside forces, portions of the memory are often erased. Therefore, words are stored redundantly in various areas of the memory. If a particular word is stored in n different places in the memory, what is the probability that this word cannot be recalled if one-half of the memory is erased by electromagnetic radiation? Hint: Consider each word to be stored in a particular cell (or box). These cells (boxes) may be located anywhere, geometrically speaking, in memory. The contents of each cell may be either erased or not erased. Assume n is small compared to the memory capacity.

8. In a certain lottery, six numbers are randomly chosen from the set {0, 1, 2 . . . 49} (without replacement). To win the lottery, a player must guess correctly all six numbers, but it is not necessary to specify in which order the numbers are selected.

   (a) What is the probability of winning the lottery with only one ticket?
   (b) Suppose in a given week, 6 million lottery tickets are sold. Suppose further that each player is equally likely to choose any of the possible number combinations and does so independently of the selections of all other players. What is the probability that exactly four players correctly select the winning combination?
   (c) Again assuming 6 million tickets sold, what is the most probable number of winning tickets?
   (d) Repeat parts (b) and (c) using the Poisson approximation to the binomial probability distribution. Is the Poisson distribution an accurate approximation in this example?

9. In a digital communication system, a block of k data bits is mapped into an n bit code word that typically contains the k information bits as well as n−k redundant bits. This is known as an (n, k) block code. The redundant bits are included to provide error correction capability. Suppose that each transmitted bit in our digital communication system is received in error with probability p. Furthermore, assume that the decoder is capable of correcting any pattern of t or fewer errors in an n bit block. That is, if t or fewer bits in an n bit block are received in error, then the code word will be decoded correctly, whereas if more than t errors occur, the decoder will decode the received word incorrectly. Assuming each bit is received in error with probability P = 0. 03 find the probability of decoder error for each of the following codes.

   (a) (n, k) = (7, 4), t = 1
   (b) (n, k) = (15, 7), t = 2
   (c) (n, k) = (31, 16), t = 3