School of Engineering & Applied Science
Ahmedabad University
MA202-Probability and Random processes
Homework Assignment -2

Submission Date : 17 February 2017

1. An experiment consists of rolling a pair of (six-sided) dice and observing the sum. This experiment is repeated until the sum of 7 is observed at which point the experiment stops. let N be the random variable which represents the number of times the experiment is repeated. That is, if the first occurrence of sum=7 happens on the 5th roll of the dice, then N=5.
   (a) Find the probability mass function for the random variable N. That is, find $P_N(k) = Pr(N = k)$for all k.
   (b) What is the probability that the experiment proceeds for at least 4 rolls? That is Pr(N $\geq$ 4)
2. A balanced coin is tossed nine times. Find the probabilities of each of the following events: (Hint : You can use discrete random variable)
   (a) exactly 3 heads occurred;
   (b) at least 3 heads occurred;
   (c) at least 3 heads and at least 2 tails occurred.
3. Cards are drawn from a standard 52-card deck until the third club is drawn. After each card is drawn, it is put back in the deck and the cards are reshuffled so that each card drawn is independent of all others. (Hint : You can use discrete random variable)
   (a) Find the probability that the 3rd club is drawn on the 8th selection.
   (b) Find the probability that at least 8 cards are drawn before the 3rd club appears.
   (c) Repeat parts (a) and (b) if the cards are drawn without replacement. That is, after each card is drawn, the card is set aside and not replaced in the deck.
4. Suppose the arrival of telephone calls at a switch can be modeled with a Poisson PMF. That is, if X is the number of calls that arrives in t minutes, then

   $$\Pr(\text{X} = \text{k}) = \frac{(\lambda t)^k}{k!} e^{-\lambda t}, k = 0, 1, 2, 3, ...$$

   where λ is the average arrival rate in calls/minute. Suppose that the average rate of calls is 10 per minute.
   (a) What is the probability that fewer than three calls will be received in the first 6 seconds?
   (b) What is the probability that fewer than three calls will be received in the first 6 minutes?
5. A certain random variable has a probability density function of the form $f_X(x) = \frac{c}{x^2+4}$. Find the following
   (a) the constant c
   (b) Pr (X $>$ 2)
   (c) Pr (X $<$ 3)
   (d) Pr (X $<$ 3 | X $>$ 2)
6. Prove the integral identity, $I = \int_{-\infty}^{\infty} exp(-\frac{x^2}{2})dx = \sqrt{2\pi}$ (Hint : It may be easier to show that $I^2 = 2\pi$)
7. Using the normalization integral for a Gaussian random variable, find an analytical expression for the following integral :

   $I = \int_{-\infty}^{\infty} exp(-(ax^2 + bx + C))dx$, where a $>$ 0 , b and c are constants