School of Engineering & Applied Science
Ahmedabad University
MA202-Probability and Random processes
Homework Assignment - 3

Submission Date : 3 March 2017

1. (a) Digital communication system sends two messages, M = 0 or M = 1, with equal probability. A receiver observes a voltage which can be modeled as a Gaussian random variable, X, whose PDFs conditioned on the transmitted message are given by
$f_{X|M=0}(x) = \frac{1}{\sqrt{2\pi\sigma^2}} exp(-\frac{x^2}{2\sigma^2})$ and $f_{X|M=1}(x) = \frac{1}{\sqrt{2\pi\sigma^2}} exp(-\frac{(x-1)^2}{2\sigma^2})$

    i. Find and Plot $Pr(M = 0|X = x)$ as a function of x for $\sigma^2 = 1$. Repeat for $\sigma^2 = 5$
    ii. Repeat part (a) assuming that the a priori probabilities are $Pr(M = 0) = 1/4$ and $Pr(M = 1) = 3/4$

   (b) In above question , suppose our receiver must observe the random variable X and then make a decision as to what message was sent. Furthermore, suppose receiver makes a three-level decision as follows:

   Decide 0 was sent if $Pr(M = 0|X = x) \geq 0.9$
   Decide 1 was sent if $Pr(M = 1|X = x) \geq 0.9$
   Erase the symbol(decide not to decide) $Pr(M = 0|X = x) < 0.9$ and $Pr(M = 1|X = x) < 0.9$

   Assuming the two messages are equally probable, $Pr(M = 0) = Pr(M = 1) = 1/2$ and that $\sigma^2 = 1$, find
    i. the range of x over which each of the three decisions should be made,
    ii. the probability that the receiver erases a symbol,
    iii. the probability that the receiver makes an error (i.e. decides a "0" as sent when a "1" was actually sent, or vice versa).

2. Mr. Hood is a good archer. He can regularly hit a target having a 3-ft. diameter and often hits the bulls-eye, which is 0.5 ft. in diameter, from 50 ft. away. Suppose the miss is measured as the radial distance from the center of the target and, further, that the radial miss distance is a Rayleigh random variable with the constant in the Rayleigh PDF being $\sigma^2 = 4(sq.ft)$

   (a) Determine the probability of Mr. Hoods hitting the target.
   (b) Determine the probability of Mr. Hoods hitting the bulls-eye.
   (c) Determine the probability of Mr. Hoods hitting the bulls-eye given that he hits the target.

3. For a Gaussian random variable, derive expressions for the coefficient of skewness and the coefficient of kurtosis in terms of the mean and variance, μ and σ^2

4. Prove that all odd central moments of a Gaussian random variable are equal to zero. Furthermore, develop an expression for all even central moments of a Gaussian random variable.

5. Use the characteristic function (or the moment generating function or the probability generating function) to show that a Poisson PMF is the limit of a binomial PMF with n approaching infinity and p approaching zero in such a way that np = μ = constant.