School of Engineering & Applied Science
Ahmedabad University
MA202-Probability and Random processes
Homework Assignment - 4

Submission Date : 24 March 2017

1. Suppose X is uniformly distributed over $(-a, a)$, where a is some positive constant. Find the PDF of $Y = X^2$.
2. Suppose X is a random variable with an exponential PDF of the form $f_X(x) = 2e^{-2x}u(x)$. A new random variable is created according to the transformation $Y = 1 - X$.
   (a) Find the range for X and Y.
   (b) $f_Y(y)$
3. For positive constants a and b, a pair of random variables has a joint PDF specified by
$$f_{X,Y}(x, y) = abe^{-(ax+by)}u(x)u(y)$$
   (a) Find the joint CDF,$f_{X,Y}(x, y)$
   (b) Find the marginal PDFs, $f_X(x)$ and $f_Y(y)$
   (c) Find $Pr(X > Y)$.
   (d) Find $Pr(X > Y^2)$.
4. For positive constants a, b, c, and positive integer n, a pair of random variables has a joint PDF specified by
$$f_{X,Y}(x, y) = \frac{d}{(ax + by + c)^n}u(x)u(y)$$
   (a) Find the constant d in terms of a, b, c, and n.
   (b) Find the marginal PDFs, $f_X(x)$ and $f_Y(y)$
   (c) Find $Pr(X > Y)$
5. A pair of random variables has a joint PDF specified by
$$f_{X,Y}(x, y) = d \cdot exp(-(ax^2 + bxy + cy^2))$$
   (a) Find the constant d in terms of a, b, c. Also, find any restrictions needed for a , b, and c themselves for this to be a valid PDF.
   (b) Find the marginal PDFs, $f_X(x)$ and $f_Y(y)$
   (c) Find $Pr(X > Y)$.
6. A quarterback throws a football at a target marked out on the ground 40 yards from his position. Assume that the PDF for the footballs hitting the target is Gaussian within the plane of the target. Let the coordinates of the plane of the target be denoted by the x and y axes. Thus, the joint PDF of (X,Y) is a two-dimensional Gaussian PDF. The average location of the hits is at the origin of the target, and the standard deviation in each direction is the same and is denoted as σ. Assuming X and Y are independent, find the probability that the hits will be located within an annular ring of width dr located a distance r from the origin; that is, find the probability density function for hits as a function of the radius from the origin.
7. Suppose X and Y are independent and exponentially distributed both with unit-mean. Consider the roots of the quadratic equation $z^2 + Xz + Y = 0$
   (a) Find the probability that the roots are real.
   (b) Find the probability that the roots are complex.
   (c) Find the probability that the roots are equal..