School of Engineering & Applied Science
Ahmedabad University
MA202-Probability and Random processes
Homework Assignment - 5

Submission Date : 18 April 2017

1. A discrete random process, X[n], is generated by repeated tosses of a coin. Let the occurrence of a head be denoted by 1 and that of a tail by -1. A new discrete random process is generated by Y[2n] = X[n] for n = 0,$\pm$1,$\pm$2,... and Y[n] = X[n+1] for n odd(either positive or negative). Find the autocorrelation function for Y[n].

2. A random process is defined by $X(t) = e^{-At}u(t)$, where A is a random variable with PDF, $f_A(a)$.

   (a) Find the PDF of $X(t)$ in terms of $f_A(a)$

   (b) If A is an exponential random variable, with $f_A(a) = e^{-a}u(a)$, find $\mu_X(t)$ and $R_{X,X}(t_1, t_2)$.

3. Let a random process consist of a sequence of pulses with the following properties: (i) the pulses are rectangular with equal duration, Δ(with no dead space in between pulses), (ii) the pulse amplitudes are equally likely to be $\pm$1, (iii) all pulse amplitudes are statistically independent, and (iv) the various members of the ensemble are not synchronized.

   (a) Find the mean function, $\mu_X(t)$.

   (b) Find the autocorrelation function,$R_{X,X}(t_1, t_2)$.

4. A workstation is used until it fails and is then sent out for repair. The time between failures, or the length of time the workstation functions until it needs repair, is a random variable T. Assume the times between failures, $T_1, T_2...., T_n$ of the workstations available are independent random variables that are identically distributed. For $t > 0$, let the number of workstations that have failed be $N(t)$.

   (a) If the time between failures of each workstation has an exponential PDF, then what type of process is $N(t)$?

   (b) Assume that you have just purchased 10 new workstations and that each has a 90-day warranty. If the mean time between failures (MTBF) is 250 days, what is the probability that at least one workstation will fail before the end of the warranty period?

5. Suppose the arrival of calls at a switchboard is modeled as a Poisson process with the rate of calls per minute being λ_a=0.1

   (a) What is the probability that the number of calls arriving in a 10-minute interval is less than 10?

   (b) What is the probability that the number of calls arriving in a 10-minute interval is less than 10 if λ_a=10?

   (c) Assuming λ_a=0.1 , what is the probability that one call arrives during the first 10-minute interval and two calls arrive during the second 10-minute interval?