CS 131 – Spring 2017 – Lab 1

Question 1 How would you write each of these propositions using combinations of e, meaning "Sue is an English major," and j, meaning "Sue is a junior" with the operations $\wedge, \vee, \neg$, and $\rightarrow$?

(a) Sue is a junior English major.

(b) Sue is either an English major or she is a junior.

(c) Sue is a junior, but she is not an English major.

(d) Sue is neither an English major nor a junior.

(e) Sue is exactly one of the following: an English major or a junior.

(f) Sue is a junior only if Sue is not an English major.

Solution

(a) $j \wedge e$

(b) $j \vee e$

(c) $j \wedge \neg e$

(d) $\neg j \wedge \neg e$

(e) $(\neg e \wedge j) \vee (e \wedge \neg j)$

(f) $j \rightarrow \neg e$

Question 2 Determine whether each of these conditional statements is true or false.

(a) If $1 + 1 = 2$, then $2 + 2 = 5$.

(b) If $1 + 1 = 3$, then $2 + 2 = 4$.

(c) If $1 + 1 = 3$, then $2 + 2 = 5$.

(d) If monkeys can fly, then $1 + 1 = 3$.

Solution

(a) This is false since 1+1 does equal 2 but 2+2 does not equal 5.

(b) This is true since 1 + 1 does not equal 3 so the implication is vacuously true.

(c) This is true for the same reason as (b).

(d) This is true since monkeys cannot fly.

Question 3 Use a truth table to establish the following logical equivalences.

(a) $\neg(\neg p \vee q) \equiv p \wedge \neg q$

(b) $p \wedge (p \vee q) \equiv p$

(c) $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

Solution

(a)

p	q	$\neg p$	$\neg q$	$\neg p \vee q$	$\neg(\neg p \vee q)$	$p \wedge \neg q$
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	T	F	F
F	F	T	T	T	F	F

$\neg(\neg p \vee q)$ *and* $p \wedge \neg q$ *are equivalent since columns 6 and 7 are the same.*

(b)

p	q	$p \vee q$	$p \wedge (p \vee q)$
T	T	T	T
T	F	T	T
F	T	T	F
F	F	F	F

$p \wedge (p \vee q)$ *and* p *are equivalent since columns 4 and 1 are the same.*

(c)

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$	$p \vee q$	$p \vee r$	$(p \vee q) \wedge (p \vee r)$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	F	T	F	F
F	F	T	F	F	F	T	F
F	F	F	F	F	F	F	F

$p \vee (q \wedge r)$ *and* $(p \vee q) \wedge (p \vee r)$ *are equivalent since columns 5 and 8 are the same.*

Question 4 You meet two inhabitants of Smullylan's Island. A says, "Exactly one of us is lying," and B says, "At least one of us is telling the truth." Who (if anyone) is telling the truth?

Solution *First we translate the statements into propositional logic. Let* p *be the statement "A is telling the truth" and* q *be the statement "B is telling the truth." "Exactly one of us is lying" is* $(p \wedge \neg q) \vee (\neg p \wedge q)$ *and "at least one of us is telling the truth" is* $p \vee q$*. Now we can consider all of the possibilities for each person's status as truth teller or liar, which is all of the possibilities for statements* p *and* q*.*

p	q	$\neg p$	$\neg q$	$p \wedge \neg q$	$\neg p \wedge q$	*"Exactly one of us is lying"* $(p \wedge \neg q) \vee (\neg p \wedge q)$	*"At least one of us is telling the truth"* $p \vee q$
T	T	F	F	F	F	F	T
T	F	F	T	T	F	T	T
F	T	T	F	F	T	T	T
F	F	T	T	F	F	F	F

Now consider the meaning of each row.
Row 1: Both A and B are telling the truth, but what A said is false. This cannot be the case.
Row 2: A is telling the truth and B is lying, but what B said is true. This cannot be the case.
Row 3: A is lying and B is telling the truth, but what A said is true. This cannot be the case.
Row 4: A and B are both lying, and both of their statements are false. This can be the case.
The only consistent option is in row 4, that both A and B are lying.

Question 5 State the converse, contrapositive, and inverse of each of these conditional statements.

(a) If it snows today, I will ski tomorrow.

(b) I come to class whenever there is going to be a quiz.

(c) A positive integer is a prime only if it has no divisors other than 1 and itself

Solution

(a) Converse: If I ski tomorrow, it will snow today.
Contrapositive: If I don't ski tomorrow, it won't snow today.
Inverse: If it doesn't snow today, I won't ski tomorrow.

(b) Converse: If I come to class, there is going to be a quiz.
Contrapositive: If I don't come to class, there isn't going to be a quiz.
Inverse: If there isn't going to be a quiz, I don't come to class.

(c) Converse: If a positive integer has no divisors other than 1 and itself, it is prime.
Contrapositive: If a positive integer has divisors other than 1 and itself, it is not prime.
Inverse: If a positive integer is not prime, it has divisors other than 1 and itself.