CS 131 – Spring 2017 – Lab 2

Question 1 Which of the following formulae are equivalent to each other?

(a) $p \to (q \to r)$

(b) $q \to (p \to r)$

(c) $(p \to q) \wedge (p \to r)$

(d) $(p \wedge q) \to r$

(e) $p \to (q \wedge r)$

Solution

(a)

p	q	r	$(q \to r)$	$p \to (q \to r)$
T	T	T	T	T
T	T	F	F	F
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	F	T
F	F	T	T	T
F	F	F	T	T

(b)

p	q	r	$p \to r$	$q \to (p \to r)$
T	T	T	T	T
T	T	F	F	F
T	F	T	T	T
T	F	F	F	T
F	T	T	T	T
F	T	F	T	T
F	F	T	T	T
F	F	F	T	T

(c)

p	q	r	$p \to q$	$p \to r$	$(p \to q) \wedge (p \to r)$
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	T	F
T	F	F	F	F	F
F	T	T	T	T	T
F	T	F	T	T	T
F	F	T	T	T	T
F	F	F	T	T	T

(d)

p	q	r	$p \wedge q$	$(p \wedge q) \rightarrow r$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	T
T	F	F	F	T
F	T	T	F	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T

(e)

p	q	r	$q \wedge r$	$p \rightarrow (q \wedge r)$
T	T	T	T	T
T	T	F	F	F
T	F	T	F	F
T	F	F	F	F
F	T	T	T	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T

As you can see from the truth tables, (a), (b) and (d) are equivalent and (c) and (e) are equivalent.

Question 2 Show that $(p \rightarrow q) \rightarrow r$ and $p \rightarrow (q \rightarrow r)$ are not logically equivalent.

Solution

Consider the truth assignment $p = F, q = T$, and $r = F$. With this truth assignment, $(p \rightarrow q)$ is true, and $(p \rightarrow q) \rightarrow r$ is false. $p \rightarrow (q \rightarrow r)$ is true, since $p = F$. Thus, the formulae are not equivalent.

Question 3 Determine the truth value of each of these statements if the domain consists of all integers, if the domain consists of all positive integers, and lastly if the domain consists of all natural numbers.

(a) $\forall n(n + 1 > n)$

(b) $\exists n(2n = 3n)$

(c) $\exists n(n = -n)$

(d) $\forall n(3n \leq 4n)$

Solution

- *All integers*
  - *(a) True*
  - *(b) True*
  - *(c) True*

(d) False

- *All positive integers*

  (a) True

  (b) False

  (c) False

  (d) True

- *All natural numbers*

  (a) True

  (b) True

  (c) True

  (d) True

Question 4 Translate each of these statements into logical expressions in three different ways by varying the domain and by using predicates with one and with two variables.

(a) A student in your school has lived in Vietnam.

(b) A student in your school knows Java, Prolog, and C++.

(c) Everyone in your class enjoys Thai food.