CS 131 – Spring 2017 – Lab 3

Question 1 Let $P(x)$ be the statement "x can speak Russian" and let $Q(x)$ be the statement "x knows the computer language C++." Express each of these sentences in terms of $P(x), Q(x)$, quantifiers, and logical connectives. The domain for quantifiers consists of all students at your school.

(a) There is a student at your school who can speak Russian and who knows C++.

(b) There is a student at your school who can speak Russian but who doesn't know C++.

(c) Every student at your school either can speak Russian or knows C++.

(d) No student at your school can speak Russian or knows C++.

Solution

(a) $\exists x(P(x) \land Q(x))$

(b) $\exists x(P(x) \land \neg Q(x))$

(c) $\forall x(P(x) \lor Q(x))$

(d) $\neg\exists x(P(x) \lor Q(x))$

Question 2 Translate each of these statements into logical expressions using predicates, quantifiers, and logical connectives.

(a) No one is perfect.

(b) Not everyone is perfect.

(c) All your friends are perfect.

(d) At least one of your friends is perfect.

Solution *Let the domain for each of the following be all people, let $F(x)$ be "x is your friend," and let $P(x)$ be "x is perfect."*

(a) $\neg\exists x(P(x))$

(b) $\neg\forall x(P(x))$

(c) $\forall x(F(x) \to P(x))$

(d) $\exists x(F(x) \land P(x))$

Question 3 Translate each of these statements into logical expressions in three different ways by varying the domain and by using predicates with one and with two variables.

(a) A student in your school has lived in Vietnam.

(b) A student in your school knows Java, Prolog, and C++.

(c) Everyone in your class enjoys Thai food.

Solution *Let $Y(x)$ be the propositional function that x is in your school and $C(x)$ be the propositional function that x is in your class.*

(a) If we let $V(x)$ be "x has lived in Vietnam," then we have $\exists x V(x)$ if the domain is just your schoolmates, or $\exists x(Y(x) \wedge V(x))$ if the domain is all people. If we let $D(x, y)$ mean that person x has lived in country y, then we can rewrite this last one as $\exists x(Y(x) \wedge D(x, \text{Vietnam}))$.

(b) If we let $J(x), P(x)$, and $C(x)$ be the propositional functions asserting x's knowledge of Java, Prolog, and C++, respectively, then we have $\exists x(J(x) \wedge P(x) \wedge C(x))$ if the domain is just your schoolmates, or $\exists x(Y(x) \wedge J(x) \wedge P(x) \wedge C(x))$ if the domain is all people. If we let $K(x, y)$ mean that person x knows programming language y, then we can rewrite this last one as $\exists x(Y(x) \wedge K(x, \text{Java}) \wedge K(x, \text{Prolog}) \wedge K(x, \text{C++}))$.

(c) If we let $T(x)$ be "x enjoys Thai food," then we have $\forall x T(x)$ if the domain is just your classmates, or $\forall x(C(x) \rightarrow T(x))$ if the domain is all people. If we let $E(x, y)$ mean that person x enjoys food of type y, then we can rewrite this last one as $\forall x(C(x) \rightarrow E(x, \text{Thai}))$.

Question 4 Translate the following definition into a logical statement using quantifiers: x and y are coprime if their only common divisor is 1.
Solution *Let $C(x, y)$ be "x and y are coprime." Then $C(x, y) := \forall a \exists b \exists c(ab = x \wedge ac = y) \rightarrow (a = 1)$.*

Question 5 Prove that x is divisible by 6 if and only if x is divisible by both 2 and 3 by translating each side into a logical statement and proving that the statements are equivalent.
Solution *Let $S(x)$ be "x is divisible by 6," and $T(x)$ be "x is divisible by two and three." Then $S(x) := \exists y(6y = x)$ and $T(x) := \exists y \exists z(2y = x \wedge 3z = x)$. Starting with the formula for $S(x)$ we transform $S(x)$ into $T(x)$:*

$$\begin{aligned}
\exists y(6y = x) &\equiv \exists y(6y = x \wedge 6y = x) && \text{idempotent law of } \wedge \\
&\equiv \exists y(2(3y) = x \wedge 3(2y) = x) && \text{associativity of multiplication} \\
&\equiv \exists y \exists z(2y = x \wedge 3z = x)) && \text{renaming}
\end{aligned}$$