Question 1 Consider the following incorrect theorem:

Incorrect Theorem. Suppose n is a natural number larger than 2, and n is not a prime number. Then 2n+13 is not a prime number.

What are the hypotheses and conclusion of this theorem? Show that the theorem is incorrect by finding a counterexample.

Question 2 Suppose a and b are real numbers. Prove that if a < b < 0 then $a^2 > b^2$.

Question 3 Suppose a, b, c, and d are real numbers, 0 < a < b, and d > 0. Prove that if $ac \ge bd$ then c > d.

Question 4 Suppose that y + x = 2y - x, and x and y are not both zero. Prove that $y \neq 0$.

Question 5 Use a direct proof to show that every odd integer is the difference of two squares.