CS 131 – Spring 2017 – Lab 6

Question 1 Theorem. All horses are the same color.

Proof. We'll induct on the number of horses. Base case: 1 horse. Clearly with just 1 horse, all horses have the same color.
Now, for the inductive step: we'll show that if it is true for any group of N horses, that all have the same color, then it is true for any group of N+1 horses. Well, given any set of N+1 horses, if you exclude the last horse, you get a set of N horses. By the inductive step these N horses all have the same color. But by excluding the first horse in the pack of N+1 horses, you can conclude that the last N horses also have the same color. Therefore all N+1 horses have the same color. What's wrong with this proof by induction? Multiple answers are allowed.

- In the base step
- In the inductive step
- In the choice of base step
- In the choice of inductive hypothesis
- This proof is correct

Question 2 Let P(n) be the statement that $1^2 + 2^2 + \; + n^2 = \frac{n(n+1)(2n+1)}{6}$ for the positive integer n.

a) What is the statement P(1)?

b) Show that P(1) is true, completing the basis step of the proof.

c) What is the inductive hypothesis?

d) What do you need to prove in the inductive step?

e) Complete the inductive step, identifying where you use the inductive hypothesis.

f) Explain why these steps show that this formula is true whenever n is a positive integer.

Question 3 Prove that $3 + 3 \cdot 5 + 3 \cdot 5^2 + ... + 3 \cdot 5^n = \frac{3(5^{n+1}-1)}{4}$ whenever n is a nonnegative integer.

Question 4 Prove that $n^2 - 1$ is divisible by 8 whenever n is an odd positive integer.