Question 1 A vending machine dispensing books of stamps accepts only one-dollar coins, $1 bills, and $5 bills.

a) Find a recurrence relation for the number of ways to deposit n dollars in the vending machine, where the order in which the coins and bills are deposited matters.

b) What are the initial conditions?

c) How many ways are there to deposit $10 for a book of stamps?

Question 2

a) Find a recurrence relation for the number of bit strings of length n that contain two consecutive 0's.

b) What are the initial conditions?

c) How many bit strings of length seven contain two consecutive 0's?

Question 3 Let $T(n) = 1 + T(\frac{n}{2})$ and $T(1) = 0$. Find a closed-form solution for $T(n)$ when n is a power of 2 and prove that it is correct.

Question 4 Let $T(n) = 3T(n-1) + 4$ and $T(0) = 5$. Find a closed-form solution for $T(n)$ and prove that it is correct.
Hint: Remember that $\sum_{j=0}^{n} ar^j = a + ar + ar^2 + ... + ar^n = \frac{ar^{n+1}-a}{r-1}$.