CS 131 – Spring 2017 – Lab 9

Question 1 How many 6-element RNA sequences of A, C, G and U

a) do not contain U?

b) end with GU?

c) start with C?

d) contain only A or U?

Solution

a) $3^6 = 729$

b) $4^4 = 256$

c) $4^5 = 1024$

d) $2^6 = 64$

Question 2 How many bit strings are there of length six or less, not counting the empty string?
Solution
$2^6 + 2^5 + 2^4 + 2^3 + 2^2 + 2^1 = 126$

Question 3 How many strings of three decimal digits

a) do not contain the same digit three times?

b) begin with an odd digit?

c) have exactly two digits that are 4s?

Solution

a) There are 10 strings which contain the same digit three times: 000, 111, ..., 999. So, there are $10^3 - 10 = 990$ *which do not contain the same digit three times.*

b) There are 5 odd digits, so there are $5 \cdot 10 \cdot 10 = 500$ *strings which begin with an odd digit.*

c) The number of ways to choose the placement of the 2 4s is C(3,2) = 3 and there are 9 digits remaining to choose for the last digit, so there are $3 \cdot 9 = 27$ *strings with exactly two 4s.*

Question 4 How many functions are there from the set $\{1, 2, ..., n\}$, where n is a positive integer, to the set $\{0, 1\}$

a) that are one-to-one?

b) that assign 0 to both 1 and n?

c) that assign 1 to exactly one of the positive integers less than n?

Solution

a) If $n \geq 3$ there are no one-to-one functions from $\{1, ..., n\}$ to $\{0, 1\}$. If $n = 1$ there are 2, and if $n = 2$ there are 2.

b) There are 2^{n-2} functions which assign 0 to both 1 and n.

c) First choose which positive integer less than n to assign 1 to, there are $n - 1$ choices for this. Then choose between 0 and 1 for n. Thus, there are $2(n - 1)$ such functions.

Question 5 How many subsets with more than two elements does a set with 100 elements have?
Solution *The total number of subsets of a set with 100 elements is 2^{100}. The number of subsets with 0 elements is 1, the number with 1 element is 100 (C(100, 1)) and the number with 2 elements is C(100, 2) = 4950. Thus, the number of subsets with more than two elements is $2^{100} - 1 - 100 - 4950 = 2^{100} - 5051$.*

Question 6 In how many different orders can five runners finish a race if no ties are allowed?
Solution *The order that the runners finish a race in matters, so the number of orders five runners can finish a race in is P(5, 5) = 5! = 120.*