**Name:**
**Collaborators:**
**Outside resources:**

Math 2106, Foundations of Mathematical Proof
HW 2 — Due January 30, 2017 (Mon)

Turn in the following problems from Hammack's book:
Chapter 4, problems 4, 20, 26
Chapter 5, problems 6, 8, 12, 18, 20, 24, 28
Chapter 6, problems 4, 6, 8, 10, 16, 18
Chapter 7, problem 28

**Additional exercises** (to be turned in)

1. Let $a \in \mathbb{Z}$. Prove that $3 \mid a^2$ if and only if $3 \mid a$. (You may use the fact that every integer can be written as exactly one of $3k$, $3k+1$, or $3k+2$ for some integer $k$.)

2. Let $a$, $b$, $c$, $d$, $x$ and $y$ be integers with $a \neq 0$ and $b \neq 0$.

   (a) If $a \mid c$, then $a^2 \mid c^2$.

   (b) If $a \mid c$ and $b \mid d$, then $ab \mid cd$.

   (c) If $a \mid c$ and $a \mid d$, then $a \mid cx + dy$.

   (d) If $a \mid b$ and $b \mid a$, then $a = b$ or $a = -b$.

   (e) If $a \nmid cd$, then $a \nmid c$ and $a \nmid d$.

3. Prove or disprove.

   (a) The sum of two rational numbers is rational.

   (b) The sum of an irrational number and a rational number is irrational.

   (c) The sum of two irrational numbers is irrational.

   (d) The product of two rational numbers is rational.

   (e) The product of an irrational number and a rational number is irrational.

   (f) The product of two irrational numbers is irrational.

4. Let $a$ be an irrational number, $s$ a real number, and $r$ a nonzero rational number. Prove that $ar + s$ or $ar - s$ is irrational.