

Quantifiers

Solutions are to be written on the board.

1. Translate these statements into English, where $C(x)$: “ x is a comedian,” $F(x)$: “ x is funny”, and the domain of discourse is all people. Try to find short sentences that sound natural in English.

(a) $\forall x (C(x) \Rightarrow F(x))$

(b) $\forall x (C(x) \wedge F(x))$

(c) $\exists x (C(x) \Rightarrow F(x))$

(d) $\exists x (C(x) \wedge F(x))$

2. Rewrite each of the following statements so that no negation is outside a quantifier or an expression involving logical connectives.

(a) $\sim \forall x \forall y P(x, y)$

(b) $\sim \forall y \exists x P(x, y)$

(c) $\sim \forall y \forall y (P(x, y) \vee Q(x, y))$

(d) $\sim (\exists x \exists y \sim P(x, y) \wedge \forall x \forall y Q(x, y))$

(e) $\sim \forall x (\exists y \forall z P(x, y, z) \wedge \exists z \forall y P(x, y, z))$

3. Let $F(x, y)$ be the predicate “ x can fool y ” where the domain of discourse is the set of all people in the world. Express each of these statements using logical quantifiers.

(a) Nobody can fool everybody.

(b) Somebody cannot be fooled by anybody.

(c) Everybody can fool somebody.

(d) There is no one who can fool everybody.

(e) Everyone can be fooled by somebody.

(f) There is somebody who can be fooled by everybody.