What is a proof?

A proof only becomes a proof after the social act of "accepting it as a proof."

Yuri Manin, mathematician and inventor of quantum computing
in *A Course in Mathematical Logic for Mathematicians*

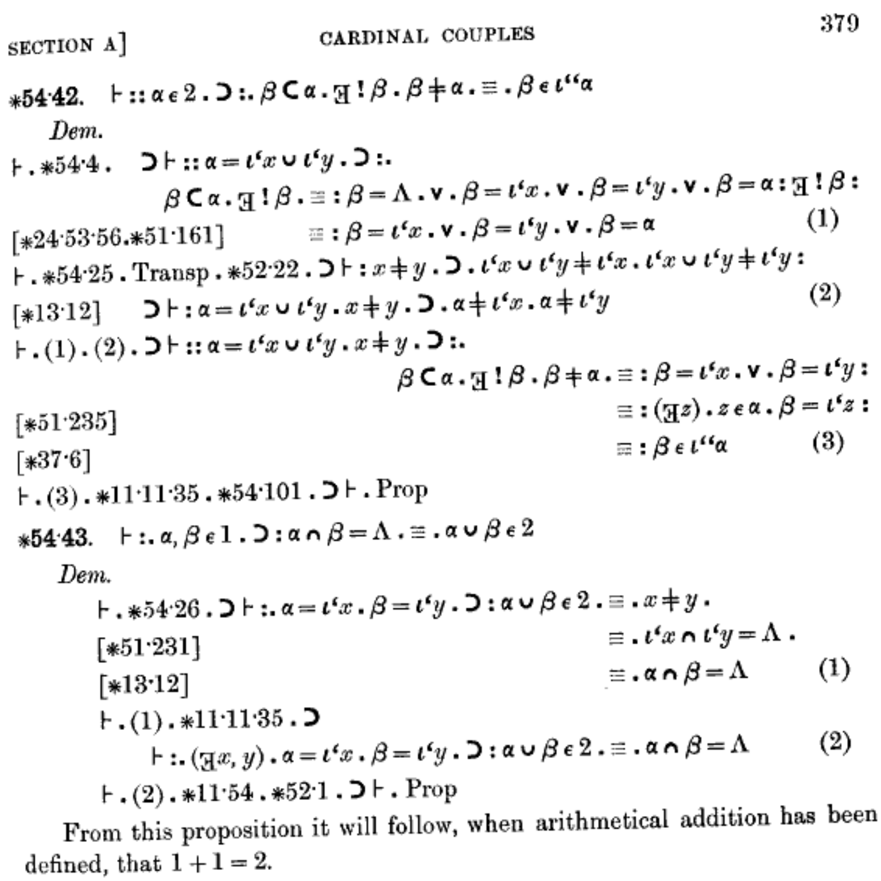
A proof of a theorem is a written verification that shows that the theorem is definitely and unequivocally true. A proof should be understandable and convincing to anyone who has the requisite background and knowledge.

Richard Hammack, in *Book of Proof*

What is a proof? The question has two answers. The right wing ("right-or-wrong", "rule-of-law") definition is that a proof is a logically correct argument that establishes the truth of a given statement. The left wing answer (fuzzy, democratic, and human centered) is that a proof is an argument that convinces a typical mathematician of the truth of a given statement.

While valid in an idealistic sense, the right wing definition of a proof has the problem that, except for trivial examples, it is not clear that anyone has ever seen such a thing.

Keith Devlin, mathematician at Stanford
in "When is a proof?"



SECTION A] CARDINAL COUPLES 379

$*54{\cdot}42.\ \vdash :: \alpha \in 2 . \supset :. \beta \subset \alpha . \exists ! \beta . \beta \neq \alpha . \equiv . \beta \in \iota``\alpha$

Dem.

$\vdash . *54{\cdot}4 . \supset \vdash :: \alpha = \iota`x \cup \iota`y . \supset :.$

$\beta \subset \alpha . \exists ! \beta . \equiv : \beta = \Lambda . \vee . \beta = \iota`x . \vee . \beta = \iota`y . \vee . \beta = \alpha : \exists ! \beta :$

$[*24{\cdot}53{\cdot}56 . *51{\cdot}161] \quad \equiv : \beta = \iota`x . \vee . \beta = \iota`y . \vee . \beta = \alpha \quad (1)$

$\vdash . *54{\cdot}25 . \text{Transp} . *52{\cdot}22 . \supset \vdash : x \neq y . \supset . \iota`x \cup \iota`y \neq \iota`x . \iota`x \cup \iota`y \neq \iota`y :$

$[*13{\cdot}12] \quad \supset \vdash : \alpha = \iota`x \cup \iota`y . x \neq y . \supset . \alpha \neq \iota`x . \alpha \neq \iota`y \quad (2)$

$\vdash . (1) . (2) . \supset \vdash :: \alpha = \iota`x \cup \iota`y . x \neq y . \supset :.$

$\beta \subset \alpha . \exists ! \beta . \beta \neq \alpha . \equiv : \beta = \iota`x . \vee . \beta = \iota`y :$

$[*51{\cdot}235] \quad \equiv : (\exists z) . z \in \alpha . \beta = \iota`z :$

$[*37{\cdot}6] \quad \equiv : \beta \in \iota``\alpha \quad (3)$

$\vdash . (3) . *11{\cdot}11{\cdot}35 . *54{\cdot}101 . \supset \vdash . \text{Prop}$

$*54{\cdot}43.\ \vdash :. \alpha, \beta \in 1 . \supset : \alpha \cap \beta = \Lambda . \equiv . \alpha \cup \beta \in 2$

Dem.

$\vdash . *54{\cdot}26 . \supset \vdash :. \alpha = \iota`x . \beta = \iota`y . \supset : \alpha \cup \beta \in 2 . \equiv . x \neq y .$

$[*51{\cdot}231] \quad \equiv . \iota`x \cap \iota`y = \Lambda .$

$[*13{\cdot}12] \quad \equiv . \alpha \cap \beta = \Lambda \quad (1)$

$\vdash . (1) . *11{\cdot}11{\cdot}35 . \supset$

$\vdash :. (\exists x, y) . \alpha = \iota`x . \beta = \iota`y . \supset : \alpha \cup \beta \in 2 . \equiv . \alpha \cap \beta = \Lambda \quad (2)$

$\vdash . (2) . *11{\cdot}54 . *52{\cdot}1 . \supset \vdash . \text{Prop}$

From this proposition it will follow, when arithmetical addition has been defined, that $1 + 1 = 2$.

From *Principia Mathematica* (Vol. 1), by Alfred Whitehead and Bertrand Russell, published in 1910, where it took over 300 pages to prove that "1+1=2".

1. **Definition:** Suppose a and b are integers. We say that a **divides** b if ______

2. **Definition:** An integer n is **even** if ______

3. **Definition:** An integer n is **odd** if ______

4. Prove that each integer is either even or odd but not both.

Write down the statements you used without proof: