Proof Practice using Even and Odd Numbers

Prove the following using the definitions of even and odd integers. You may use the fact that each integer is either even or odd but not both. Two integers are said to have the same parity if they are both even or they are both odd.

1. Prove that x and x^2 have the same parity for any integer x.
2. Prove that two integer x and y have the same parity only if $x + y$ is even.
3. Prove that two integer x and y have the same parity only if $x - y$ is even.
4. Prove that two integer x and y have the same parity if $x + y$ is even.
5. Prove that two integer x and y have the same parity if $x - y$ is even.
6. Prove that $x + y$ and $x - y$ have the same parity for any two integers x and y.