Which of the following proofs is the most valid?
Find all the errors.

THEOREM: *For any positive integer n, if n^2 is a multiple of 3, then n is a multiple of 3.*

A.

PROOF. Assume that n^2 is an odd positive integer that is divisible by 3. That is $n^2 = (3n+1)^2 = 9n^2 + 6n + 1 = 3n(n+2) + 1$. Therefore, n^2 is divisible by 3. Assume that n^2 is even and a multiple of 3. That is, $n^2 = (3n)^2 = 9n^2 = 3n(3n)$. Therefore, n^2 is a multiple of 3. If we factor $n^2 = 9n^2$, we get $3n(3n)$; which means that n is a multiple of 3. ■

B.

PROOF. Suppose to the contrary that n is not a multiple of 3. We will let $3k$ be a positive integer that is a multiple of 3, so that $3k + 1$ and $3k + 2$ are integers that are not multiples of 3. Now $n^2 = (3k+1)^2 = 9k^2 + 6k + 1 = 3(3k^2 + 2k) + 1$. Since $3(3k^2 + 2k)$ is a multiple of 3, $3(3k^2 + 2k) + 1$ is not. Now we will do the other possibility, $3k + 2$. So, $n^2 = (3k+2)^2 = 9k^2 + 12k + 4 = 3(3k^2 + 4k + 1) + 1$ is not a multiple of 3. Because n^2 is not a multiple of 3, we have a contradiction. ■

C.

PROOF. Let n be an integer such that $n^2 = 3x$ where x is an integer. Then $3 | n^2$. Since $n^2 = 3x$, $nn = 3x$. Thus, $3|n$. Therefore if n^2 is a multiple of 3, then n is a multiple of 3. ■

D.

PROOF. Let n be a positive integer such that n^2 is a multiple of 3. Then $n = 3m$ where $m \in Z^+$. So $n^2 = (3m)^2 = 9m^2 = 3(3m^2)$. This breaks down into $3m$ times $3m$ which shows that m is a multiple of 3. ■

Write your own proof:

Reference: Annie Selden and John Seldon. "Validation of Proofs Considered as Texts". Journal for Research in Mathematics Education 2003, Vol 34, No. 1, 4-36.