Name: ______________________

Math 2106 – Exam 1
September 16, 2016

page	1	2	3	4	writing	total
points						
maximum	15	20	20	20	5	80

1. (1 point) Take a deep breath. Slowly write "I got this!": ______________________

2. (14 points) Let P and Q be statements. Consider the following compound statements:

$$
\begin{array}{ccccc}
P \wedge Q, & P \wedge \sim Q\ , & \sim P \wedge Q\ , & \sim P \wedge \sim Q, & P \wedge \sim P, \\
P \vee Q\ , & P \vee \sim Q\ , & \sim P \vee Q\ , & \sim P \vee \sim Q, & P \vee \sim P, \\
P \Rightarrow Q, & \sim P \Rightarrow Q, & \sim Q \Rightarrow P, & \sim P \Rightarrow \sim Q, & \sim Q \Rightarrow \sim P,
\end{array}
$$
$$
[P \wedge (P \Rightarrow Q)] \Rightarrow Q, \qquad [Q \wedge (P \Rightarrow Q)] \Rightarrow P,
$$

(a) Find all statements on the list that are logically equivalent to each of the following. No explanation is needed.

- $P \Rightarrow Q$

- $\sim (P \Rightarrow Q)$

- $P \vee Q$

(b) Find all tautologies on the list and explain why each of them is a tautology. If you use truth tables, show intermediate steps.

3. (20 points) For each of the following statements, (i) write it in symbols, (ii) write its negation in symbols so that $\sim$ does not appear outside of quantifiers, (iii) prove or disprove the original statement (clearly indicate which).

   (a) Every real number is less than or equal to its square.

   (b) The sum of two integers is never equal to their product.

   (c) There exists a non-zero integer whose cube equals its negative.

4. (20 points) Prove or disprove the following statements. You may assume the definition of odd and even integers. You may assume that an integer is even if and only if it is not odd. You may assume that the sum or product of two integers is again an integer. If you use any other statement, you must prove it here first.

   (a) For every integer a, the numbers a and $(a+1)(a-1)$ have opposite parity.

   (b) For any integers a and b, if ab is even, then a is even or b is even.

5. (20 points) Prove or disprove each of the following using only the definitions.

   (a) The real number $\sqrt{2} + 1$ is irrational.

   (b) The sum of two irrational numbers is irrational.

   (c) The product of two irrational numbers is irrational.