

Name: _____

Math 2106 – Exam 2
October 14, 2016

Write clearly in complete sentences using correct terminology. Solutions must be readable from left to right and from top to bottom.

page	1	2	3	4	writing	total
points						
maximum	10	10	10	10	2	42

1. Prove each of the following using only the definitions. Let A, B and C be sets.

(a) $A \subseteq B \iff \mathcal{P}(A) \subseteq \mathcal{P}(B)$

(b) $(A - B) \cap (A - C) = A - (B \cup C)$

2. Let d be a positive integer and a and b be integers such that $d \mid (a - b)$.
Prove that $d \mid (a^n - b^n)$ for all integers $n \geq 0$.
Do not use the fact that multiplication is well defined on $\mathbb{Z}_d$ unless you prove it first.
Hint (if you need help with algebra): $ac - bd = (ac - bc) + (bc - bd)$.

3. Determine whether each of the following relations on $\mathbb{R}$ is an equivalence relation. Justify your answers.
- (a) $x R y$ if $x - y = 2\pi n$ for some integer n
 - (b) $x R y$ if $x + y = 2\pi n$ for some integer n

4. Let a and b be positive integers. Prove that $a \mid b$ if and only if $[x]_b \subseteq [x]_a$ for all $x \in \mathbb{Z}$. Recall that $[x]_n = \{y \in \mathbb{Z} : y \equiv x \pmod{n}\}$.