More proofs, quantifiers

1. Let x and y be integers. Prove that the following are equivalent.
   (a) x and y have the same parity.
   (b) $x + y$ is even.
   (c) $x - y$ is even.
2. Prove that $x^4 - 4x^3 + 1 = 0$ for some real number x.
3. Find a statement $P(x, y)$ where $x \in S$, $y \in T$ such that $\forall x \in S, \exists y \in T, P(x, y)$ is true, but $\exists y \in T, \forall x \in S, P(x, y)$ is false.
4. Do the same as above, but with "true" and "false" swapped.