Exam 1 review

A1 List important logical equivalences.

A2 List important rules of inference (implications that are tautologies).

A3 Show how to negate statements involving $\wedge$, $\vee$, $\Rightarrow$, $\Leftrightarrow$, $\forall$, $\exists$.

A4 Explain why a direct proof works.

A5 Explain why a contrapositive proof works.

A6 Explain why a proof by contradiction works.

Prove or disprove each of the following statements.

B1 There is no smallest positive irrational number.

B2 There is a rational number a and an irrational number b such that a^b is rational.

B3 There is an integer n such that $n + n^2 + n^3 + n^4$ is odd.

B4 The sum of squares of two odd integers cannot be the square of an integer.

B5 For any integer n, $5 \mid n$ if and only if $5 \mid n^2$.

B6 For any integer n, $12 \mid n$ if and only if $12 \mid n^2$.