Name:
Collaborators:
Outside resources:

Math 2106, Foundations of Mathematical Proof
HW 3 — Due February 15, 2017 (Wed)

Turn in the following problems from Hammack's book:
Chapter 8, problems 2, 8, 12, 14, 18, 20, 22, 24, 28.
Chapter 9, problems 6, 8, 14, 16, 26, 34.

Additional exercises (to be turned in)

1. Let A and B be sets. Prove that $A \cup B = (A - B) \cup (B - A) \cup (A \cap B)$.
2. Prove or disprove each of the following.
   (a) For any sets A, B, and C, if $A \cap B = A \cap C$, then $B = C$.
   (b) For any sets A, B, and C, if $A \cup B = A \cup C$, then $B = C$.
   (c) For any sets A, B, and C, if $A \cap B = A \cap C$ and $A \cup B = A \cup C$, then $B = C$.
3. Let A and B be sets. Disprove each of the following.
   (a) $\mathcal{P}(A \setminus B) \subseteq \mathcal{P}(A) \setminus \mathcal{P}(B)$.
   (b) $\mathcal{P}(A) \setminus \mathcal{P}(B) \subseteq \mathcal{P}(A \setminus B)$.
   (c) $\mathcal{P}(A \cup B) \subseteq \mathcal{P}(A) \cup \mathcal{P}(B)$.
4. Let A, B, and C be sets. Prove that $(A \cap B) \cup C = A \cap (B \cup C)$ if and only if $C \subseteq A$.
5. Give a precise meaning to the statement "There is no Universe" and prove it from the axioms of set theory.
6. Let A be a set. Determine whether each of the following is or is not a set, according to the axioms of set theory (from Halmos' book or from in-class worksheets). Justify your answers.
   (a) $\{\{\varnothing, X\} : X \in A\}$
   (b) $\{X : X \text{ is a non-empty set }\}$
   (c) $\{\mathcal{P}(X) : X \subset A\}$