

Induction

1. Prove that the sequence $\sqrt{2}, \sqrt{2 + \sqrt{2}}, \sqrt{2 + \sqrt{2 + \sqrt{2}}}, \dots$ is strictly increasing.
2. Prove that the sequence $\sqrt{2}, \sqrt{2 + \sqrt{2}}, \sqrt{2 + \sqrt{2 + \sqrt{2}}}, \dots$ is bounded from above.
3. Prove that every natural number greater than 1 is either prime or a product of prime numbers.
4. Suppose the plane $\mathbb{R}^2$ is cut into regions by n infinitely long straight lines, where $n \geq 1$.
 - (a) The number of regions is $\leq$ _____. Prove that this is correct.
 - (b) Show that there exists an arrangement of lines that gives exactly this maximum number of regions.
5. Prove that you can cut a square into n smaller squares for all integers $n \geq 6$. (This is a homework problem.)
6. Describe the number of ways to tile a rectangle of size $2 \times n$ by dominos of size 2×1 . Prove that your answer is correct.