Relations

A **relation** R on a set A is _____

Properties of a relation R on A:

(R) **reflexive**: _____

(S) **symmetric**: _____

(T) **transitive**: _____

(A) **antisymmetric**: aRb and bRa only if $a = b$, for any $a, b \in A$.

For each of the following relations, determine which of the four properties above it satisfies.

1. $\{(1,1), (2,2), (3,3), (4,4), (1,2), (2,3), (3,4)\} \subseteq \{1,2,3,4\} \times \{1,2,3,4\}$

2. $\{(A, B) \in \mathcal{P}(S) \times \mathcal{P}(S) : |A| = |B|\}$ where S is a set

3. $\{(A, B) \in \mathcal{P}(S) \times \mathcal{P}(S) : |A| \leq |B|\}$ where S is a set

4. $\{(A, B) \in \mathcal{P}(S) \times \mathcal{P}(S) : A \subseteq B\}$ where S is a set

5. $\{(x, y) \in \mathbb{Z} \times \mathbb{Z} : x - y \text{ is even}\}$

6. $\{(x, y) \in \mathbb{Z} \times \mathbb{Z} : x - y \text{ is odd}\}$

7. $\{(x, y) \in \mathbb{R} \times \mathbb{R} : x - y \in \mathbb{Q}$

8. $\{(x, y) \in \mathbb{R} \times \mathbb{R} : x - y \in \mathbb{Z}$

9. $\{(x, y) \in \mathbb{R} \times \mathbb{R} : x - y \in \mathbb{N}$

10. $\{(x, y) \in \mathbb{R} \times \mathbb{R} : x + y \in \mathbb{Z}$

11. $\{(x, y) \in \mathbb{R} \times \mathbb{R} : x - y = 0\}$

12. $\{(x, y) \in \mathbb{R} \times \mathbb{R} : x - y \geq 0\}$

13. $\{(x, y) \in \mathbb{R} \times \mathbb{R} : xy \geq 0\}$

14. $\{(x, y) \in \mathbb{R} \times \mathbb{R} : xy > 0\} \cup \{(0,0)\}$

A relation is called an **equivalence relation** if it is reflexive, symmetric, and transitive.
It is called a **partial order** if it is reflexive, antisymmetric, and transitive.