

**Name:**

**Collaborators:**

**Outside resources:**

Math 2106, Foundations of Mathematical Proof  
HW 5 — Due March 15, 2017 (Wednesday)

From the textbook:

12.1, problems 6, 8, 10, 12.

12.2, problems 6, 10.

12.4, problems 8, 10.

12.5, problems 4, 8.

**Additional problems**

A1 Prove that the function  $f : \mathbb{N} \times \mathbb{N} \rightarrow \mathbb{N}$  given by  $f(m, n) = 2^{m-1}(2n - 1)$  is bijective.  
(Let  $\mathbb{N} = \{1, 2, 3, \dots\}$ .)

A2 Determine whether each of the following relations from  $\mathbb{Z}_6$  to  $\mathbb{Z}_9$  is a function. Justify your answers.

(a)  $\{([x]_6, [x]_9) \in \mathbb{Z}_6 \times \mathbb{Z}_9 : x \in \mathbb{Z}\}$

(b)  $\{([x]_6, [2x]_9) \in \mathbb{Z}_6 \times \mathbb{Z}_9 : x \in \mathbb{Z}\}$

(c)  $\{([x]_6, [3x]_9) \in \mathbb{Z}_6 \times \mathbb{Z}_9 : x \in \mathbb{Z}\}$

A3 Let  $f$  be a function from  $A$  and  $B$ . Let  $C \subseteq A$  and  $D \subseteq B$ .

(a) Prove or disprove:  $f^{-1}(f(C)) \subseteq C$ .

(b) Prove or disprove:  $f^{-1}(f(C)) \supseteq C$ .

(c) Prove or disprove:  $f(f^{-1}(D)) \subseteq D$ .

(d) Prove or disprove:  $f(f^{-1}(D)) \supseteq D$ .

(e) Prove that  $f$  is injective if and only if  $f^{-1}(f(C)) = C$  for all  $C \subseteq A$ .

(f) Prove that  $f$  is surjective if and only if  $f(f^{-1}(D)) = D$  for all  $D \subseteq B$ .