Functions

1. Fill in the blanks, using symbolic logic if you can.

   (a) A relation f from A to B is called a *function* if _____

   (b) A function f from A to B is called *injective* (or one-to-one) if _____

   (c) A function f from A to B is called *surjective* (or onto) if _____

   Notes:

   (a) The term "one-to-one" is misleading because every function sends each element to exactly one element. Injective functions should really be called "two-to-two".
   (b) Being surjective depends on the choice of codomain, which should be given in the description of the function.

2. Determine whether each of the following relations is a function.

   (a) $\{(x, y) \in \mathbb{Z} \times \mathbb{Z} : y = 2x\}$
   (b) $\{(x, y) \in \mathbb{Q} \times \mathbb{Q} : y = 2x\}$
   (c) $\{(x, y) \in \mathbb{Z} \times \mathbb{Z} : x = 2y\}$
   (d) $\{(x, y) \in \mathbb{Q} \times \mathbb{Q} : x = 2y\}$
   (e) $\{(x, y) \in \mathbb{R} \times \mathbb{R} : x = y^2\}$
   (f) $\{(x, y) \in \mathbb{R} \times \mathbb{R} : y = x^2\}$

3. For each of the following functions determine if it is injective, surjective, both, or neither.

   (a) $\{(x, y) \in \mathbb{Z} \times \mathbb{Z} : y + 3 = 2x\}$
   (b) $\{(x, y) \in \mathbb{Q} \times \mathbb{Q} : 3y = x + 2\}$
   (c) $\{(x, y) \in \mathbb{Z} \times \mathbb{Z} : y = |x|\}$
   (d) $\{(x, y) \in \mathbb{N} \times \mathbb{Z} : y = |x|\}$ where $\mathbb{N} = \{0, 1, 2, \dots\}$
   (e) $\{(x, y) \in \mathbb{Z} \times \mathbb{N} : y = |x|\}$ where $\mathbb{N} = \{0, 1, 2, \dots\}$
   (f) $\{(x, y) \in \mathbb{N} \times \mathbb{N} : y = |x|\}$ where $\mathbb{N} = \{0, 1, 2, \dots\}$
   (g) The tangent function from the real interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ to $\mathbb{R}$.

4. Find a bijection from $\mathbb{R}$ to the interval $(-1, 1)$ (or the other way around).

5. (Challenging problem)
   Is there a bijection between the closed interval $[-1, 1]$ and the open interval $(-1, 1)$?