Functions (part 2)

1. Let $f : A \to B$ and $g : B \to C$. Prove or give counterexamples.
   (a) If f and g are both injective, then so is $g \circ f$.
   (b) If f and g are both surjective, then so is $g \circ f$.
   (c) If $g \circ f$ is injective, then so is f.
   (d) If $g \circ f$ is injective, then so is g.
   (e) If $g \circ f$ is surjective, then so is f.
   (f) If $g \circ f$ is surjective, then so is g.

2. Let $f : A \to B$ be a function. Prove that following are logically equivalent.
   (i) The function f is bijective.
   (ii) The relation f^{-1} is a function from B to A.
   (iii) There exists a function $g : B \to A$ such that $g \circ f = i_A$ and $f \circ g = i_B$.

3. Prove that if f is a bijective function, then f^{-1} is also a bijective function.

4. Let f and g be both functions from a set A to itself such that $g \circ f$ is the identity function i_A on A. Does $f \circ g$ also have to be the identity function?