Name:
Collaborators:
Outside resources:

Math 2106, Foundations of Mathematical Proof
HW 6 — Due April 3, 2017 (Monday)

From Hammack's book:
13.1: 14.
13.2: 4, 6, 12.
13.3: 2, 4, 6, 8, 10.
13.4: 2, 4.

From Ross' book:
7.2, 7.4,
8.2(e), 8.4, 8.6, 8.8(b), 8.9.

Additional problems

A1 For each of the following sets, determine whether it is countable. Justify your answers.

(a) The set of all subsets of $\mathbb{N}$.
(b) The set of finite subsets of $\mathbb{N}$.
(c) The set of all squences of natural numbers.
(d) The set of sequences of natural numbers with finitely many non-zero entries.

A2 (a) Prove that $|\mathbb{R}^{\mathbb{R}}| \leq |\mathcal{P}(\mathbb{R} \times \mathbb{R})|$.
(b) Prove that $|\{0,1\}^{\mathbb{R}}| \leq |\mathbb{R}^{\mathbb{R}}|$.
(c) Prove that $|\mathbb{R}^{\mathbb{R}}| = |\mathcal{P}(\mathbb{R})|$.
(d) Prove that $|\mathbb{N}^{\mathbb{R}}| = |\mathcal{P}(\mathbb{R})|$.

Recall that B^A denotes the set of all functions from A to B.

A3 Let $\mathbb{I}$ be the set of irrational numbers, i.e. $\mathbb{I} = \mathbb{R} - \mathbb{Q}$.

(a) Prove that $\mathbb{I}$ is uncountable.
(b) Prove that $|\mathbb{I}| = |\mathbb{R}|$ without using the continuum hypothesis. That is, do not assume that any uncountable subset of $\mathbb{R}$ has the same cardinality has $\mathbb{R}$.