Sequences

1. Prove that the sequence $(0, 1, 0, 1, 0, 1, \dots)$ diverges.

2. Prove that the limit of a sequence is unique, if it exists.

3. Suppose $s_n \to s$ and $a < s < b$. Prove that there are only finitely many $n \in \mathbb{N}$ for wich s_n lies outside the interval (a, b).

4. Prove that convergent sequences are bounded. That is, if (s_n) converges, then there is a real number M such that $|s_n| \leq M$ for all $n \in \mathbb{N}$.

5. Prove that if (s_n) and (t_n) both converge, then $(s_n + t_n)$ also converges. Is the converse true?

6. Prove that if (s_n) and (t_n) both converge, then $(s_n t_n)$ also converges. Is the converse true?