

Exam 3 review
Functions, Cardinality, Sequences

Know how to:

- prove or disprove that a function is injective / surjective / bijective
 - prove or disprove statements about images and preimages of functions (e.g. “ $\subseteq$ ” relations between them)
 - prove or disprove that $|A| \leq |B|$, $|A| < |B|$, $|A| = |B|$
 - prove that a given sequence converges or diverges
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1. Let f be a function from a set A to a set B . Let C and D be subsets of B . Prove from the definitions that if $C \cap D = \emptyset$, then $f^{-1}(C) \cap f^{-1}(D) = \emptyset$. If f is _____, then the converse is true. (Prove it!)
2. Let f be a function from a set A to a set B . Let U and V be subsets of A . Prove from the definitions that if $f(U) \cap f(V) = \emptyset$, then $U \cap V = \emptyset$. If f is _____, then the converse is true. (Prove it!)
3. Prove that $|\mathbb{R}| = |\mathbb{R} - \{0\}|$ by finding an explicit bijection.
4. Prove that $|\mathbb{R}| = |\{0, 1, 2\}^{\mathbb{N}}|$. Clearly state all the theorems you use.
5. Let A be the set of 2×2 real matrices with determinant 1. Prove that $|A| = |\mathbb{R}|$.
6. Prove from the definitions, that the sequence $(-1)^n \frac{n}{n+1}$ diverges.