Dominators, Loop Detection, and SSA

Reminders

- Assignment 1 resubmit due on D2L by Thursday 11:59pm (tomorrow)
- Office hours today 3:30-4:30
- Reading assignment for next week has been posted, quiz questions will follow on piazza.

Last time

- Finishing up lattice-theoretic framework for data-flow analysis
- Control-flow analysis

Today

- Loops
- Identifying loops using dominators

Loop Concepts

Loop: Strongly connected subgraph of CFG with a single entry point (header)

Loop entry edge: Source not in loop & target in loop

Loop exit edge: Source in loop & target not in loop

Loop header node: Target of loop entry edge. Dominates all nodes in loop.

Back edge: Target is loop header & source is in the loop

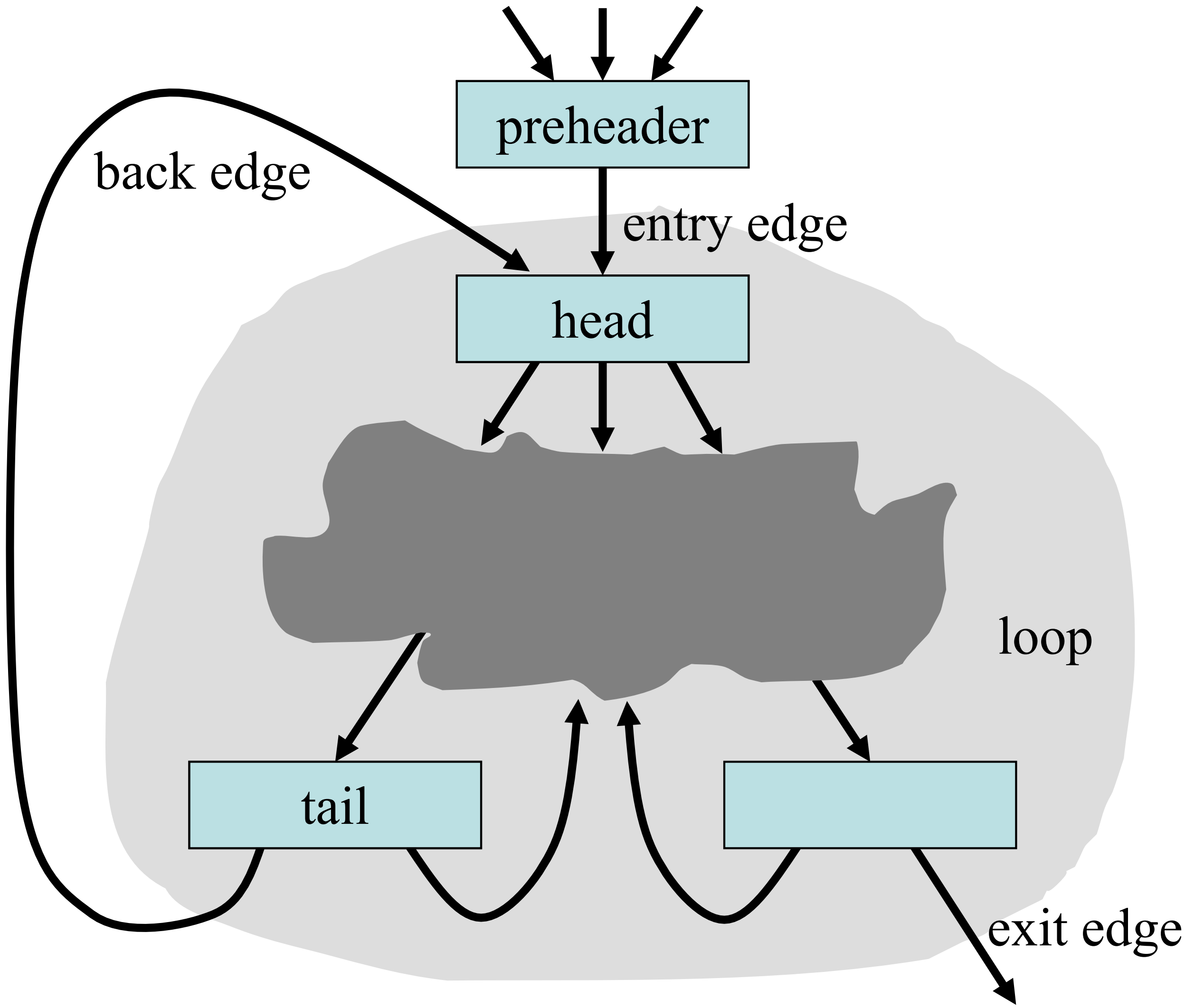
Natural loop: Associated with each back edge. Nodes dominated by header and with path to back edge without going through header

Loop tail node: Source of back edge

Loop preheader node: Single node that's source of the loop entry edge

Nested loop: Loop whose header is inside another loop

Picturing Loop Terminology



The Value of Preheader Nodes

Not all loops have preheaders

– Sometimes it is useful to create them

Without preheader node

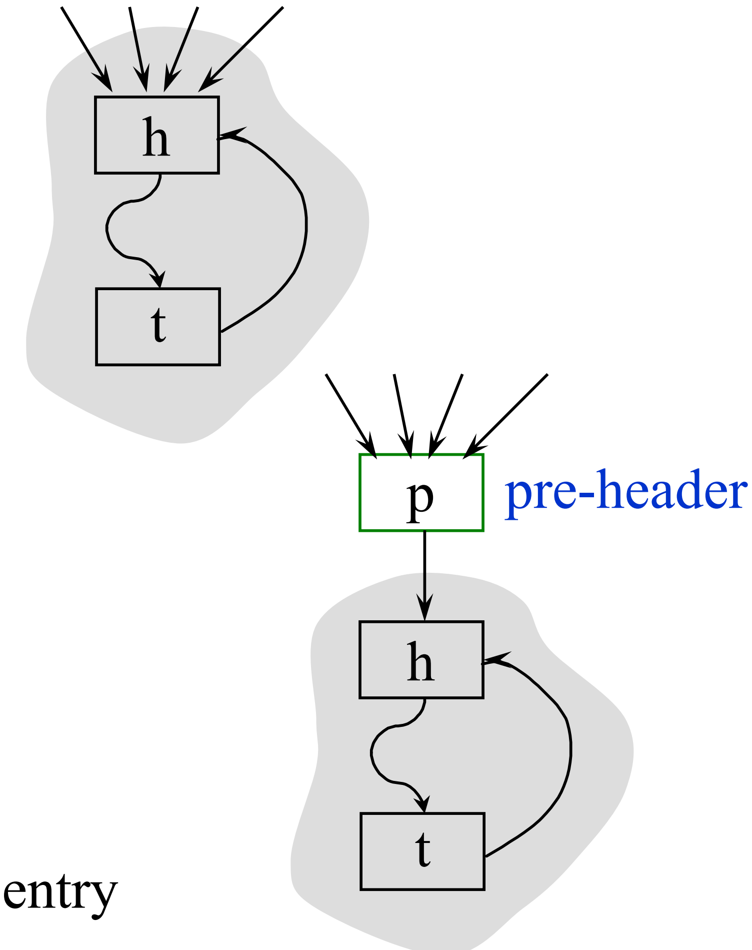
– There can be multiple entry edges

With single preheader node

– There is only one entry edge

Useful when moving code outside the loop

– Don't have to replicate code for multiple entry edges



Identifying Loops

Why?

- Most execution time spent in loops, so optimizing loops will often give most benefit

Many approaches

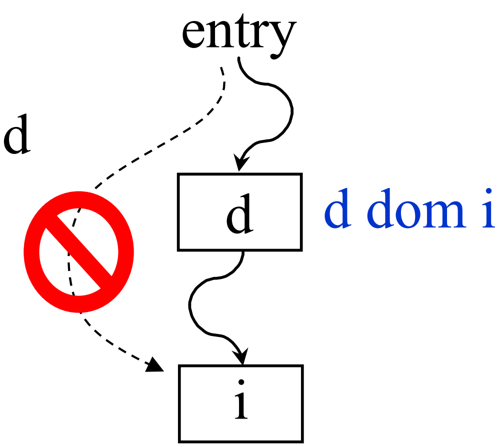
- Interval analysis
  - Exploit the natural hierarchical structure of programs
  - Decompose the program into nested regions called intervals
- Structural analysis: a generalization of interval analysis
- Identify **dominators** to discover loops

We'll focus on the dominator-based approach

Dominator Terminology

Dominators

d **dom** i if all paths from entry to node i include d

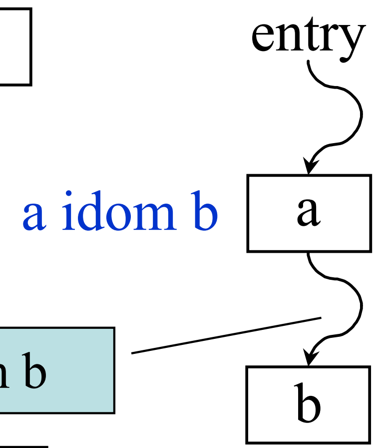


Strict dominators

d **sdom** i if d dom i and d $\neq$ i

Immediate dominators

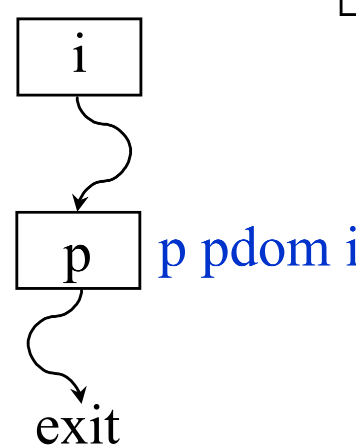
a **idom** b if a **sdom** b and there does not exist a node c such that c $\neq$ a, c $\neq$ b, a dom c, and c dom b



not $\exists$ c, a sdom c and c sdom b

Post dominators

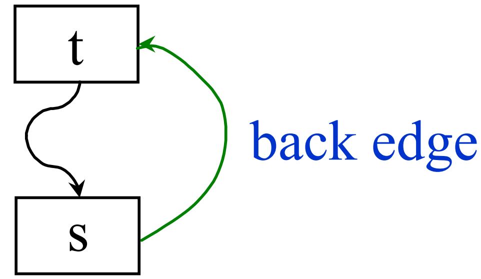
p **pdom** i if every possible path from i to exit includes p (p dom i in the flow graph whose arcs are reversed and entry and exit are interchanged)



Identifying Natural Loops with Dominators

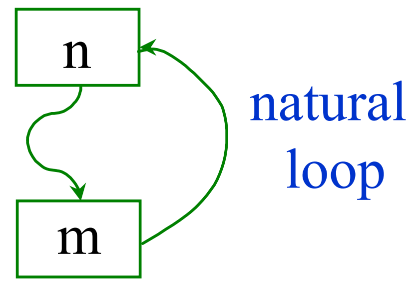
Back edges

A **back edge** of a natural loop is one whose target dominates its source



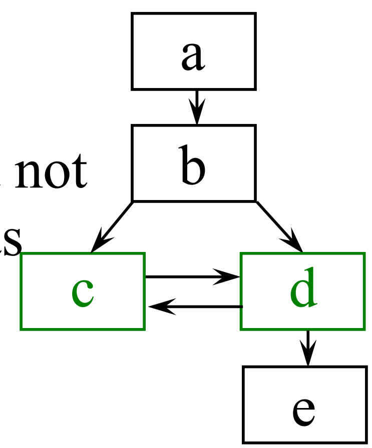
Natural loop

The **natural loop** of a back edge (m→n), where n dominates m, is the set of nodes x such that n dominates x and there is a path from x to m not containing n

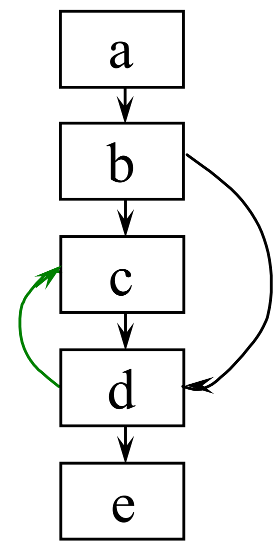


Example

SCC with c and d not a loop because has two entry points



The target, c, of the edge (d→c) does not dominate its source, d, so (d→c) does not define a natural loop



Computing Dominators

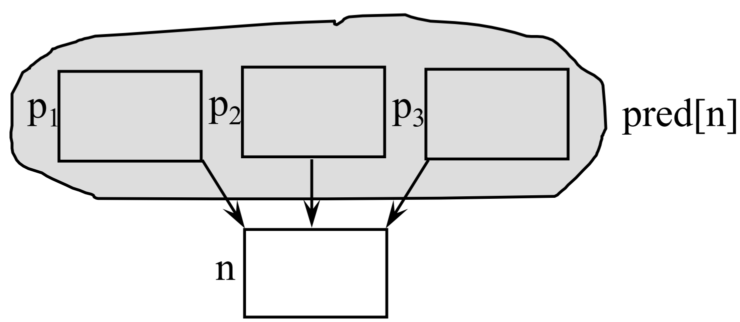
Input: Set of nodes N (in CFG), CFG, and an entry node s

Output: Dom[i] = set of all nodes that dominate node i

```
Dom[s] = {s}
for each n ∈ N – {s}
    Dom[n] = N
repeat
    change = false
    for each n ∈ N – {s}
        D = {n} ∪ (∩_{p∈pred(n)} Dom[p])
        if D ≠ Dom[n]
            change = true
            Dom[n] = D
until !change
```

Key Idea

If a node dominates all predecessors of node n, then it also dominates node n

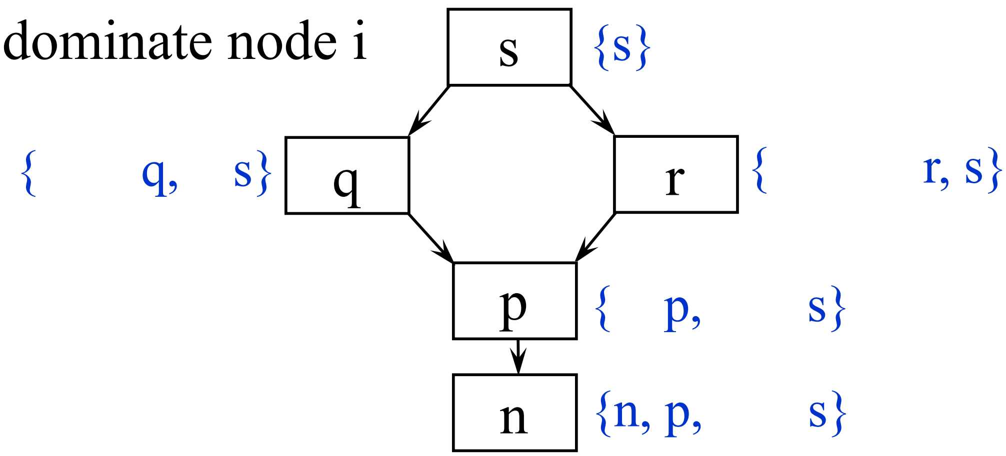


$x \in Dom(p_1) \wedge x \in Dom(p_2) \wedge x \in Dom(p_3) \Rightarrow x \in Dom(n)$

Computing Dominators (example)

Input: Set of nodes N and an entry node s

Output: Dom[i] = set of all nodes that dominate node i



Dom[s] = {s}

for each n $\in$ N – {s}

    Dom[n] = N

repeat

    change = false

    for each n $\in$ N – {s}

        D = {n} $\cup$ ($\cap_{p \in pred(n)}$ Dom[p])

        if D $\neq$ Dom[n]

            change = true

            Dom[n] = D

until !change

Initially

Dom[s] = {s}

Dom[q] = {n, p, q, r, s}. . .

Finally

Dom[q] = {q, s}

Dom[r] = {r, s}

Dom[p] = {p, s}

Dom[n] = {n, p, s}

Recall SSA, Another use of dominator information

Advantage

- Allow analyses and transformations to be simpler & more efficient/effective

Disadvantage

- May not be "executable" (requires extra translations to and from)
- May be expensive (in terms of time or space)

Process

Static Single Assignment (SSA) Form

Idea

- Each variable has only one static definition
- Makes it easier to reason about values instead of variables
- Similar to the notion of functional programming

Transformation to SSA

- Rename each definition
- Rename all uses reached by that assignment

Example

```
v := ...
... := ... v ...
v := ...
... := ... v ...
```

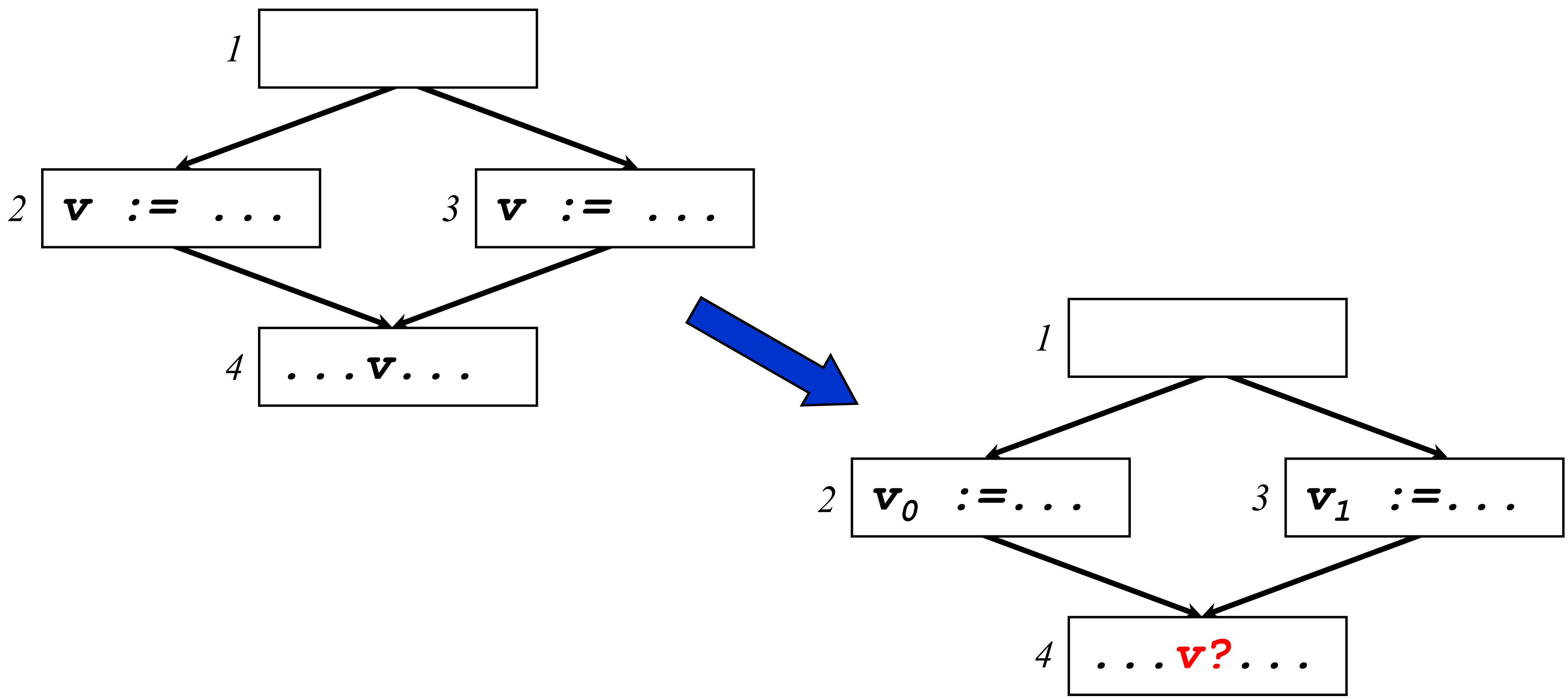
v_0 := ...
... := ... v_0 ...
v_1 := ...
... := ... v_1 ...

What do we do when there's control flow?

SSA and Control Flow

Problem

– A use may be reached by several definitions

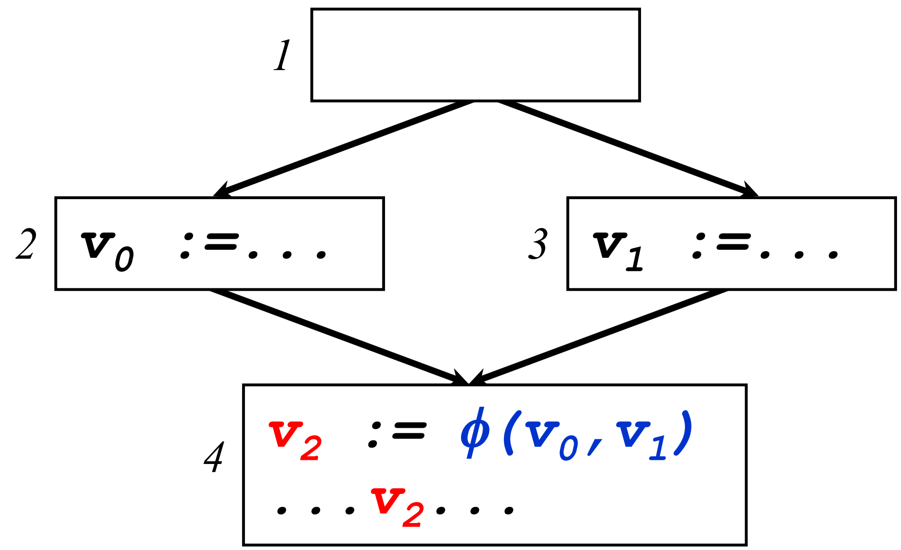


SSA and Control Flow (cont)

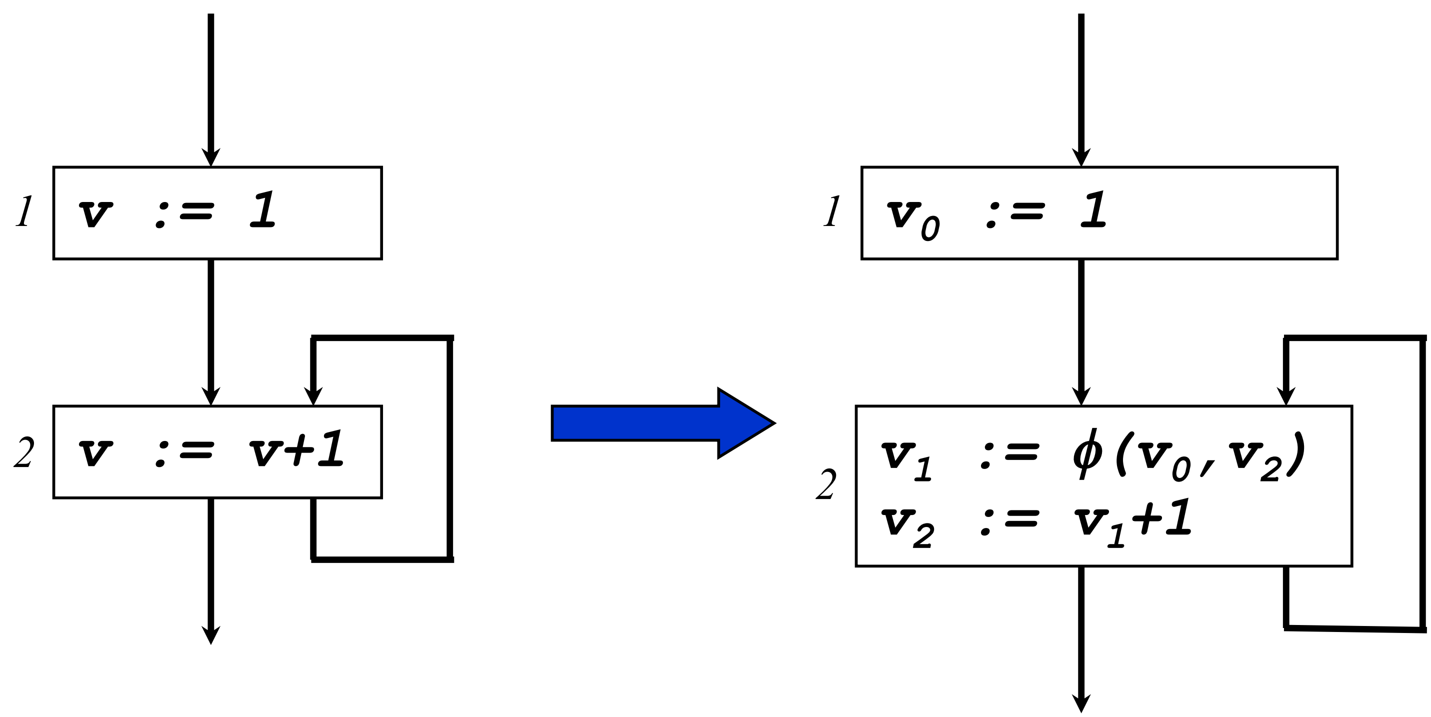
Merging Definitions

– ϕ-functions merge multiple reaching definitions

Example



Another Example



Transformation to SSA Form

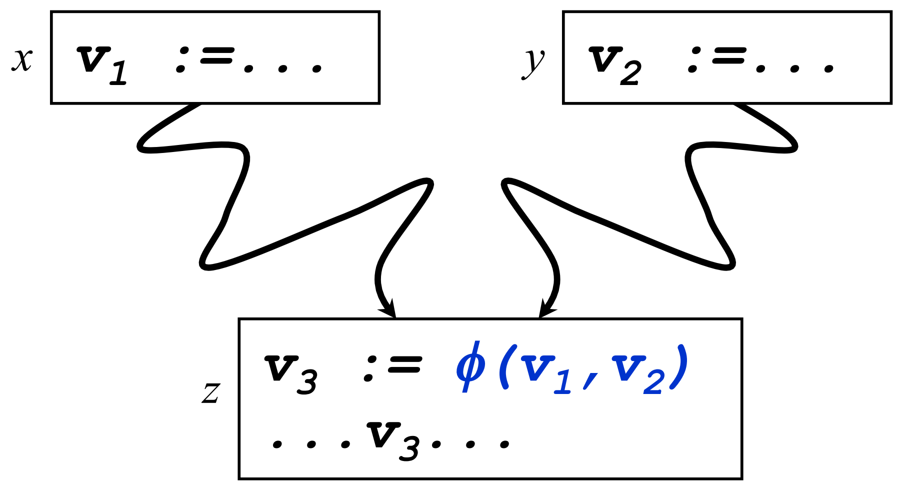
Two steps

- Insert ϕ-functions
- Rename variables

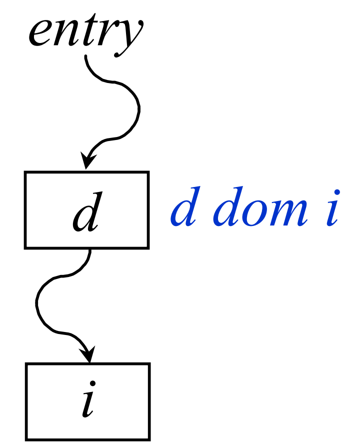
Where Do We Place ϕ-Functions?

Basic Rule

– If two distinct (non-null) paths $x \rightarrow z$ and $y \rightarrow z$ converge at node z, and nodes x and y contain definitions of variable v, then a ϕ-function for v is inserted at z



Machinery for Placing ϕ-Functions



Recall Dominators

– d **dom** i if all paths from entry to node i include d

– d **sdom** i if d dom i and d≠i

Dominance Frontiers

– The **dominance frontier** of a node d is the set of nodes that are "just barely" not dominated by d; i.e., the set of nodes n, such that

  – d dominates a predecessor p of n, and

  – d does **not** strictly dominate n

– DF(d) = {n | ∃p∈pred(n), d dom p and d !sdom n}

Notational Convenience

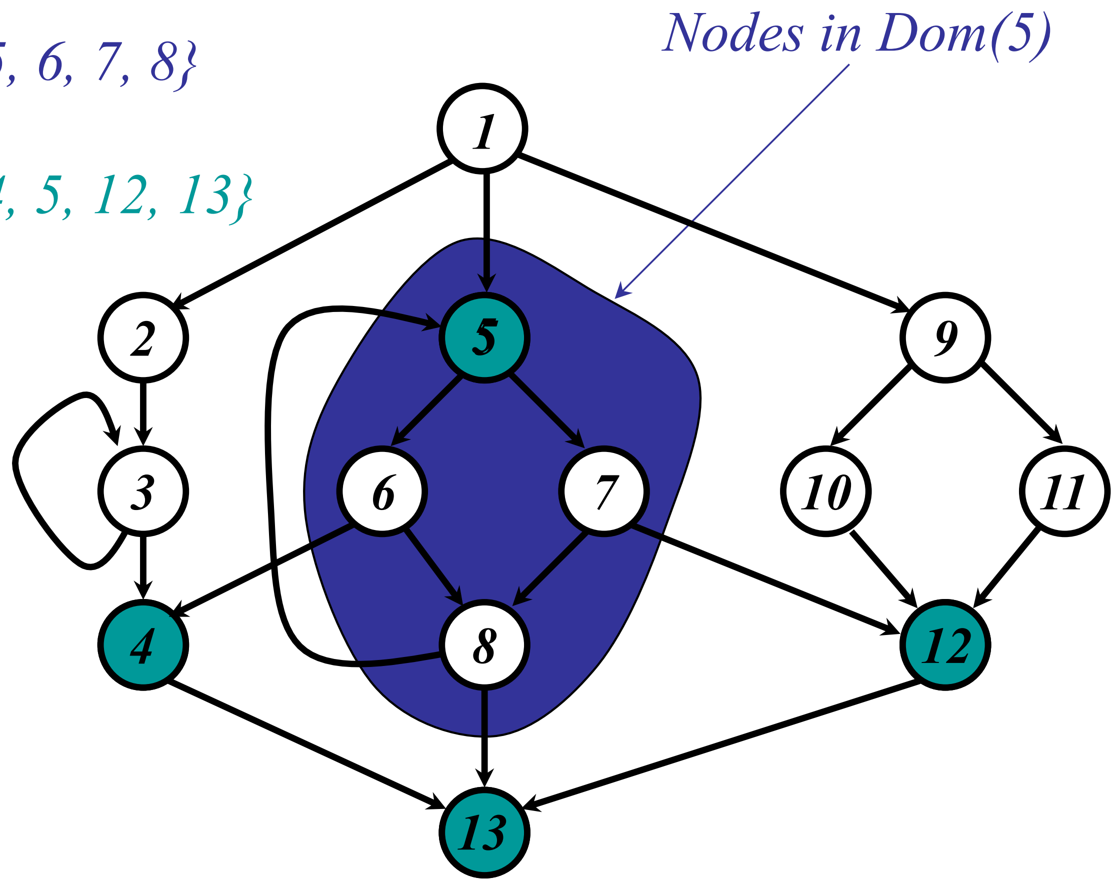
– $DF(S) = \cup_{n \in S} DF(n)$

Dominance Frontier Example

$DF(d) = \{n \mid \exists p \in pred(n),\ d\ dom\ p \text{ and } d\ !sdom\ n\}$

$Dom(5) = \{5, 6, 7, 8\}$

$DF(5) = \{4, 5, 12, 13\}$



What's significant about the Dominance Frontier?

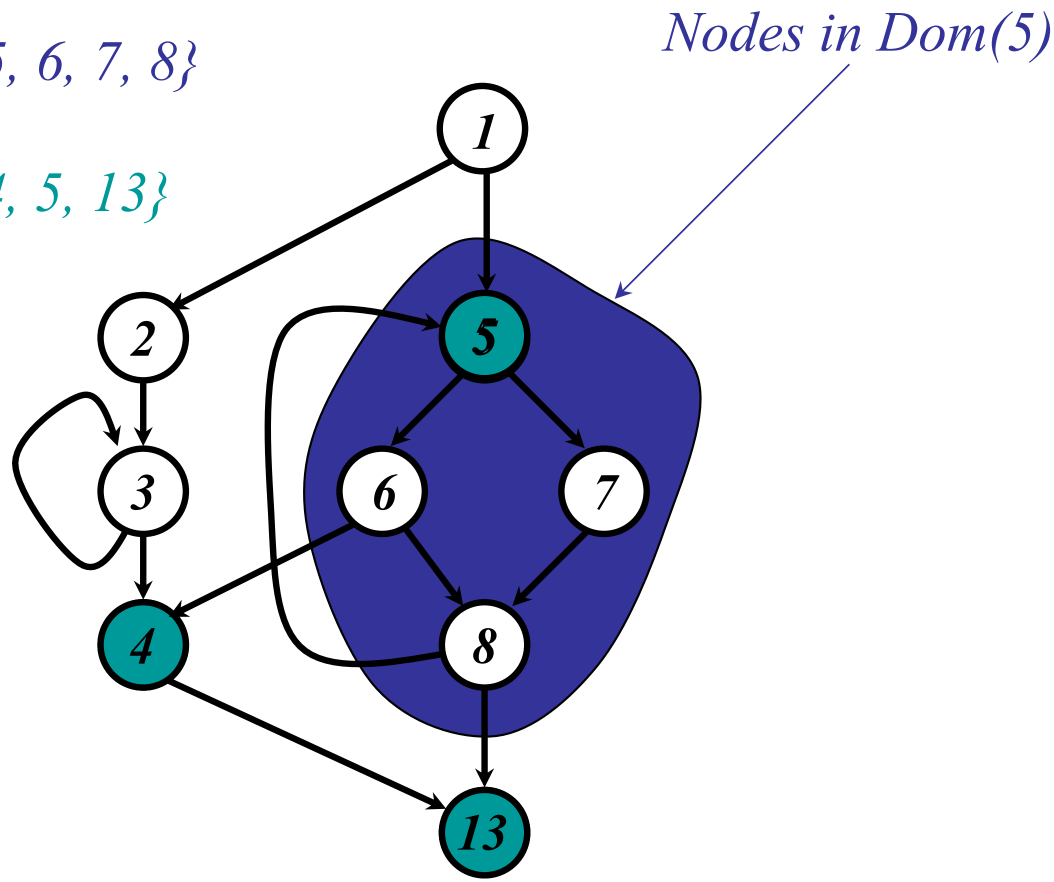
In SSA form, definitions must dominate uses

Dominance Frontier Example II

DF(d) = {n | ∃p ∈ pred(n), d dom p and d !sdom n}

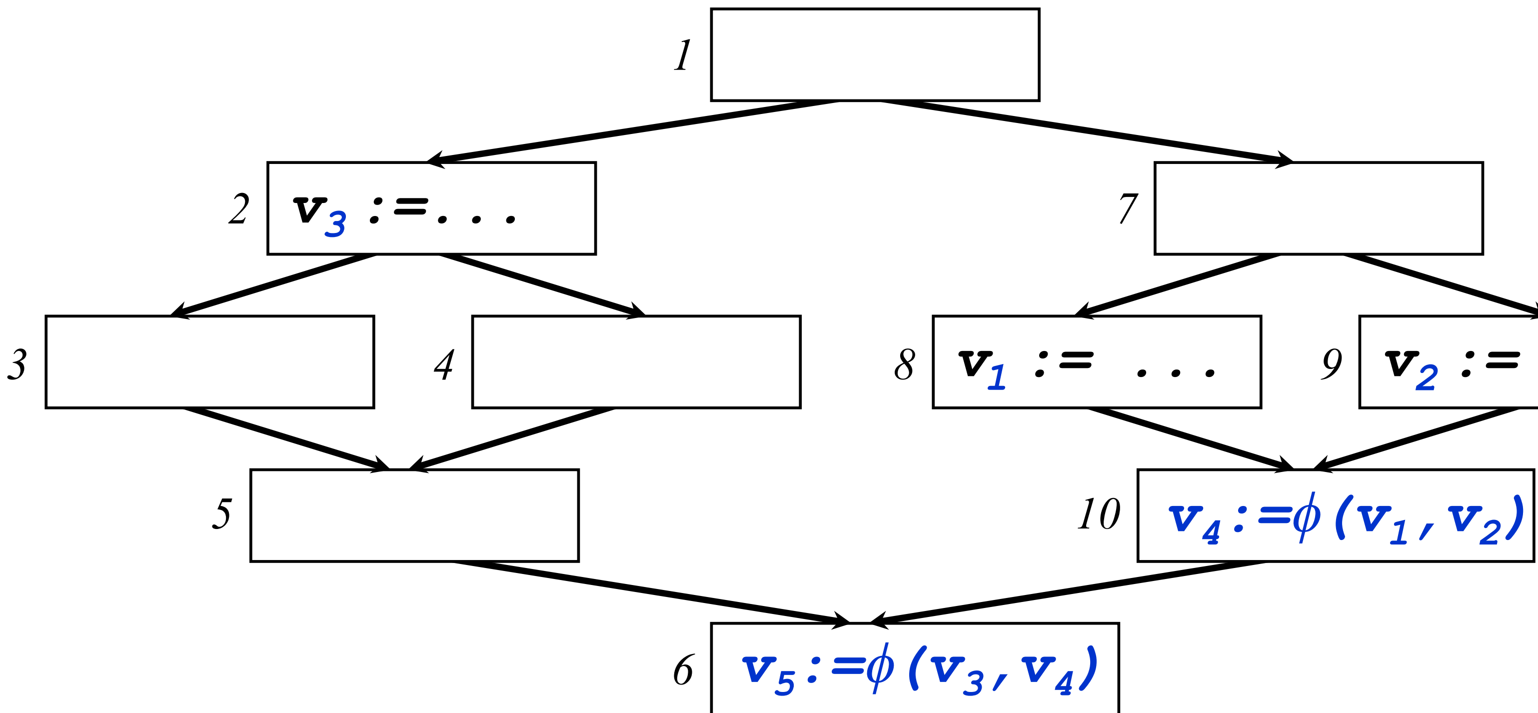
Dom(5) = {5, 6, 7, 8}

DF(5) = {4, 5, 13}



In this graph, node 4 is the first point of convergence between the entry and node 5, so do we need a ϕ- function at node 13?

SSA Exercise



DF(8) = {10}

DF(9) = {10}

DF(2) = {6}

$DF(d) = \{n \mid \exists p \in pred(n), d \text{ dom } p \text{ and } d \text{ !sdom } n\}$

DF({8,9}) = {10}

DF(10) = {6}

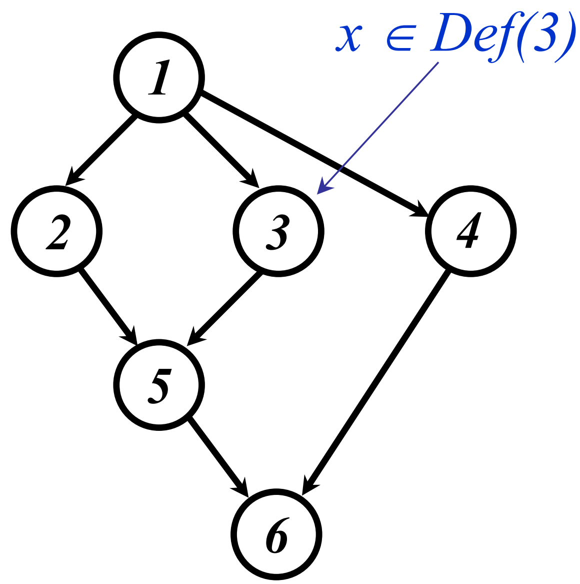
DF({2,8,9,6,10}) = {6,10}

See http://www.hipersoft.rice.edu/grads/publications/dom14.pdf for a more thorough description of DF.

Dominance Frontiers Revisited

Suppose that node 3 defines variable x

DF(3) = {5}



Do we need to insert a ϕ- function for x anywhere else?

Yes. At node 6. Why?

Dominance Frontiers and SSA

Let

- $DF_1(S) = DF(S)$
- $DF_{i+1}(S) = DF(S \cup DF_i(S))$

Iterated Dominance Frontier

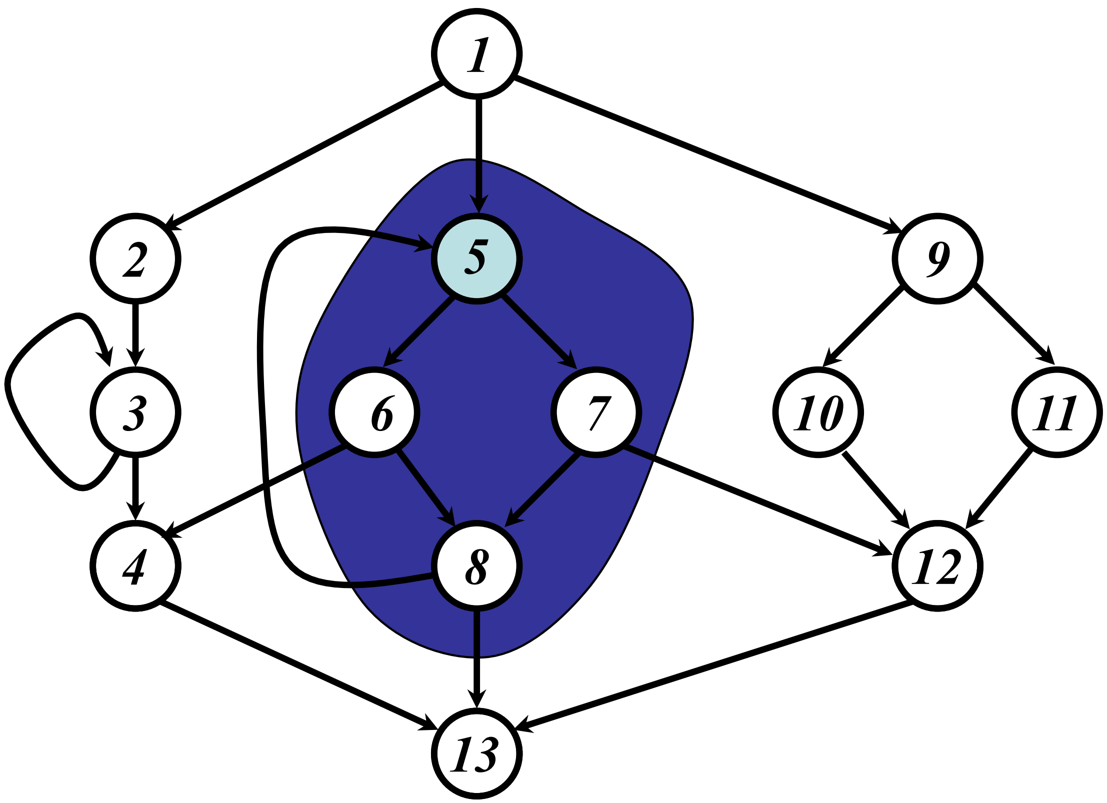
- $DF_\infty(S)$

Theorem

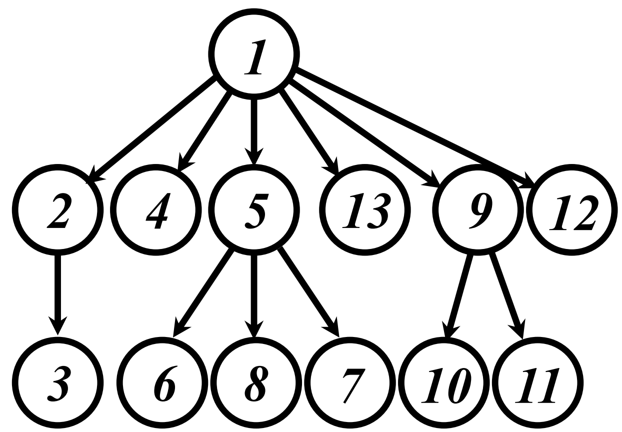
- If S is the set of CFG nodes that define variable v, then $DF_\infty(S)$ is the set of nodes that require ϕ-functions for v

Dominance Tree Example

The dominance tree shows the dominance relation



CFG



Dominance Tree

Inserting Phi Nodes

Calculate the dominator tree

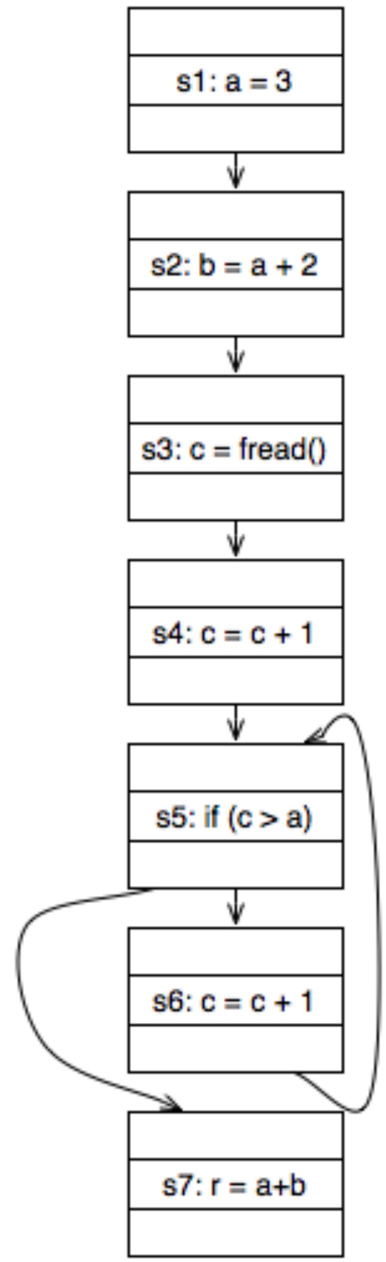
– a lot of research has gone into calculating this quickly

Computing dominance frontier from dominator tree

– $DF_{local}[n]$= successors of n (in CFG) that are not strictly dominated by n

– $DF_{up}[n]$= nodes in the dominance frontier of n that are not strictly dominated by n's immediate dominator

– $$DF[n] = DF_{local}[n] \cup \bigcup_{c \in children[n]} DF_{up}[c]$$



Algorithm for Inserting ϕ-Functions

for each variable v
    WorkList ← ∅
    EverOnWorkList ← ∅
    AlreadyHasPhiFunc ← ∅
    for each node n containing an assignment to v   *Put all defs of v on the worklist*
        WorkList ← WorkList ∪ {n}
    EverOnWorkList ← WorkList
    while WorkList ≠ ∅
        Remove some node n for WorkList
        for each d ∈ DF(n)
            if d ∉ AlreadyHasPhiFunc   *Insert at most one ϕ function per node*
                Insert a ϕ-function for v at d
                AlreadyHasPhiFunc ← AlreadyHasPhiFunc ∪ {d}
                if d ∉ EverOnWorkList   *Process each node at most once*
                    WorkList ← WorkList ∪ {d}
                    EverOnWorkList ← EverOnWorkList ∪ {d}

Transformation to SSA Form

Two steps

– Insert ϕ-functions

– Rename variables

Variable Renaming

Basic idea

- When we see a variable on the LHS, create a new name for it
- When we see a variable on the RHS, use appropriate subscript

Easy for straightline code

$$\begin{array}{l} x = \\ \quad = x \\ x = \\ \quad = x \end{array} \quad \Rightarrow \quad \begin{array}{l} x_0 = \\ \quad = x_0 \\ x_1 = \\ \quad = x_1 \end{array}$$

Use a stack when there's control flow

- *For each use of x, find the definition of x that dominates it*

$$\begin{array}{l} x = \\ \searrow \\ \quad = x \end{array} \quad \Rightarrow \quad \begin{array}{l} x_0 = \\ \searrow \\ \quad = x_0 \end{array}$$

Traverse the dominance tree

Variable Renaming (cont)

Data Structures

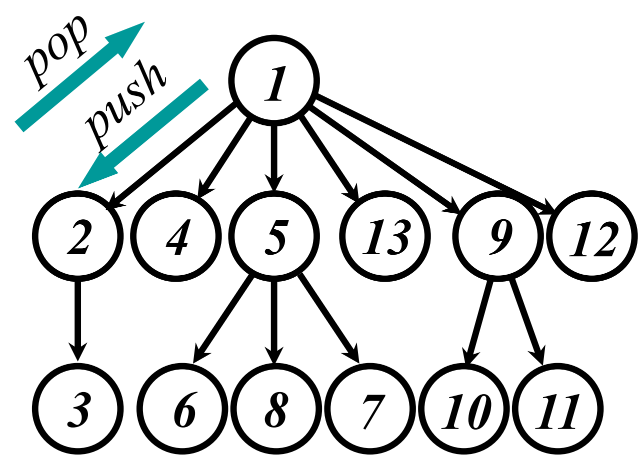
– Stacks[v] ∀v
  Holds the subscript of most recent definition of variable v, initially empty

– Counters[v] ∀v
  Holds the current number of assignments to variable v; initially 0

Auxiliary Routine

procedure GenName(variable v)

```
i := Counters[v]
push i onto Stacks[v]
Counters[v] := i + 1
```



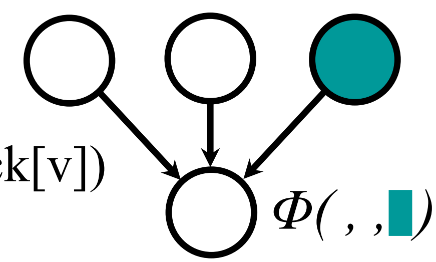
Use the Dominance Tree to remember the most recent definition of each variable

Variable Renaming Algorithm

Call Rename(entry-node)

procedure Rename(block b)
    if b previously visited return

    for each statement s in b (in order)
        for each variable $v \in$ RHS(s) (except for ϕ-functions)
            replace v by v_i, where i = Top(Stacks[v])
        for each variable $v \in$ LHS(s)
            GenName(v) and replace v with v_i, where i=Top(Stack[v])
    for each $s \in$ succ(b) (in CFG)
        $j \leftarrow$ position in s' s ϕ-function corresponding to block b
        for each ϕ-function p in s
            replace the j^{th} operand of RHS(p) by v_i, where i = Top(Stack[v])



    for each $s \in$ child(b) (in DT)
        Rename(s)

Recurse using Depth First Search

    for each ϕ-function or statement t in b
        for each $v_i \in$ LHS(t)
            Pop(Stack[v])

Unwind stack when done with this node

Transformation from SSA Form

Proposal

– Restore original variable names (*i.e.*, drop subscripts)

– Delete all ϕ-functions

Complications (the proposal doesn't work!)

– *What if versions get out of order? (simultaneously live ranges)*

$$
\begin{array}{l}
x_0 = \\
x_1 = \\
\quad\quad = x_0 \\
\quad\quad = x_1
\end{array}
$$

Alternative

– *Perform dead code elimination (to prune ϕ-functions)*

– *Replace ϕ-functions with copies in predecessors*

– *Rely on register allocation coalescing to remove unnecessary copies*

Next Time

Reading

- Advanced Compiler Optimizations for Supercomputers by Padua and Wolfe

Lecture

- Dependencies in loops
- Parallelization and Performance Optimization of Applications