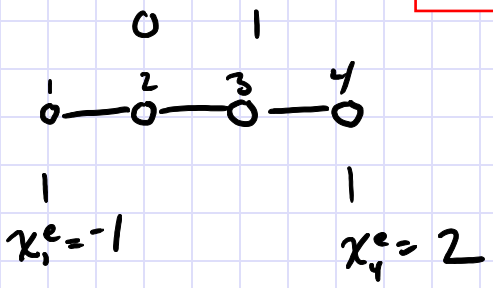
Example 1:
Compute shape functions, displacement, strain for a cubic (4-node) element.

4.1



a) Construct shape functions in terms of x

$$N_1^e = \frac{(x-x_2^e)(x-x_3^e)(x-x_4^e)}{(x_1^e-x_2^e)(x_1^e-x_3^e)(x_1^e-x_4^e)} = \frac{-x(x-1)(x-2)}{6}$$

$$N_2^e = \frac{(x-1)(x+1)(x-2)}{2}$$

$$N_3^e = \frac{-(x+1)(x)(x-2)}{2}$$

$$N_4^e = \frac{(x-x_1^e)(x-x_2^e)(x-x_3^e)}{(x_4-x_1)(x_4-x_2)(x_4-x_3)} = \frac{(x+1)\,x\,(x-1)}{6}$$

(3) (2) (1)

b) $\underset{\sim}{d}^e = 10^{-3}\begin{bmatrix}1\\0\\1\\4\end{bmatrix}$ then what is $u^e(x)$

$$10^{-3}\left(\frac{-x(x-1)(x-2)}{6} - \frac{(x+1)\,x\,(x-2)}{2} + \frac{4}{6}(x+1)(x)(x-1)\right)$$

$$-\frac{(x^3 - 3x^2 + 2x)}{6} - \frac{(x^3 - 1x^2 - 2x)}{2} + \frac{4}{6}(x^3 - x)$$

$$u^h(x) = 10^{-3} x^2$$

$$u^h(-1) = 10^{-3} \times 1$$

$$u^h(0) = 0$$

$$u^h(1) = 1$$

$$u^h(2) = 10^{-3}(4)$$

c) What is the B matrix?

$$B_3 = \frac{dN_3}{dx}$$

$$N_1 = \frac{-x(x-1)(x-2)}{6} = \frac{-x^3 + 3x^2 - 2x}{6}$$

$$B_1 = \frac{-3x^2 + 6x - 2}{6} = -\frac{1}{2}x^2 + x - \frac{1}{3}$$

$$B_2 = \frac{3}{2}x^2 - 2x - \frac{1}{2}$$

$$B_3 = -\frac{3}{2}x^2 + x + 1$$

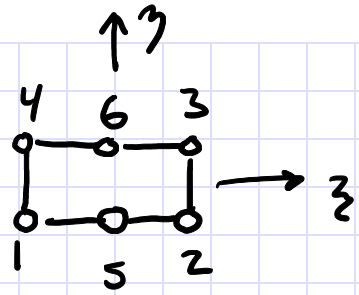
$$N_4 = \frac{(x+1)\,x\,(x-1)}{6} = \frac{x^3 - x}{6}$$

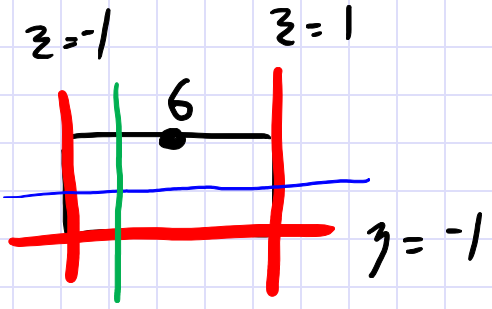
$$B_4 = \frac{1}{2}x^2 - \frac{1}{6}$$

$$\epsilon(x) = \underset{\sim}{B}^T \underset{\sim}{d}^e = \left[\left(\cancel{-\frac{1}{2}x^2} + x - \frac{1}{3}\right) + \left(\cancel{-\frac{3}{2}x^2} + x + 1\right) + 4\left(\cancel{\frac{1}{2}x^2} - \frac{1}{6}\right)\right] 10^{-3}$$

$$= 2 \times 10^{-3}\, x \quad \checkmark$$

e) what is the strain when $\underset{\sim}{d}^T = [1\ 1\ 1\ 1]$





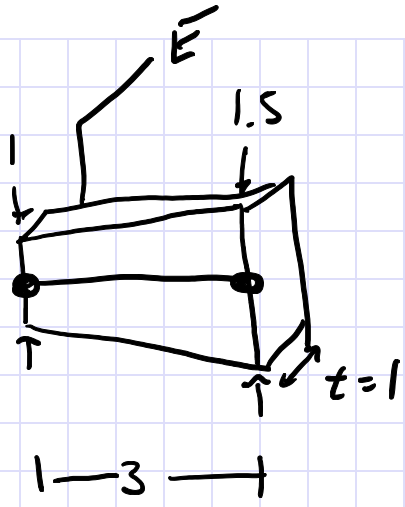
$$N_6(\xi,\eta) = -\frac{(\xi+1)(\xi-1)(\eta+1)}{2}$$

$\xi = 0$
$\eta = 1$

$$N_6(\vec{\square}_6) = 1$$

Example 2:
Determine shape function of a 6-node (mixed quadratic/linear) 2D element.

Example 3:
Compute element stiffness matrix for 1D element w/ linearly varying cross sectional area.



what is $\underset{\sim}{K}^e$?

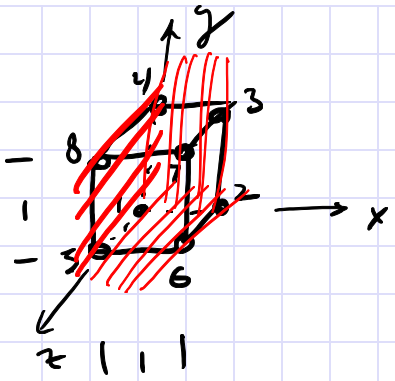
$$J^e = \frac{2x}{2\xi} = \frac{\Delta x}{\Delta \xi} = \frac{3}{2}$$

$$K^e = \int_{-1}^{1} \underset{\sim}{B}^T E A(x) \underset{\sim}{B}^e J^e \, d\xi$$

$$\underset{\sim}{B}^e = \begin{bmatrix} \frac{-1}{3} & \frac{1}{3} \end{bmatrix}$$

$$E \begin{bmatrix} \frac{-1}{3} \\ \frac{1}{3} \end{bmatrix} (1.25) \begin{bmatrix} \frac{-1}{3} & \frac{1}{3} \end{bmatrix} \frac{3}{2} (2)$$

$$= \frac{5E}{12} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$



Example 4
Write the shape function for node 7

what is

$x_7 = 1 \quad y_7 = 1 \quad z_7 = 1$

$N_7(x,y,z) = xyz$

$\vec{x} = (x,y,z)$